Appendix to Investment-Specific Technological Change and Asset Prices

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Contents

1 Solution to the Baseline model

1.1 Setup

The consumption sector uses Capital (K_C) and Labor (L_C) to produce the consumption good according to the following technology:

$$Y_C = X K_C^{\beta_C} L_C^{1-\beta_C},$$

where

$$dX_t = \mu_X \, X_t \, dt + \sigma_X X_t dB_t^x.$$

The investment sector also uses capital and labor (K_I) and (L_I) to produce the investment good

$$Y_I = X^\alpha Z_I K_I^{\beta_I} L_I^{1-\beta_I}$$

The parameter α controls the correlation between the productivity shocks in the I and C sector, and

$$dZ_{I,t} = \mu_z \, Z_{I,t} \, dt + \sigma_{1,Z} Z_{I,t} dB_t^{1,z}$$

Investing in either entails some costs of adjustment. Increasing the capital stock by I costs a total of $Z_m^{-1} \, c(I/K)K$ units of the investment good, where

$$c(i) = \frac{1}{\lambda}(1+i)^\lambda - \frac{1}{\lambda}$$

the function $c(i)$ is the cost of investment plus the adjustment costs. It satisfies $c(0) = 0$ and $c'(0) = 1$, and

$$dZ_{m,t} = \sigma_{Z,2} Z_{m,t} dB_t^{1,z}.$$

Households supply $1 - N_t$ units of labor that can be freely allocated between the two sectors,

$$L_{C,t} + L_{I,t} = 1 - N_t. \tag{1}$$

Households have preference over consumption, and leisure of the EZ form:

$$h(C, N, J) = \frac{\rho}{1-\theta^{-1}} \left(\frac{(C N^\psi)^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)\, J \right) \tag{2}$$

Here ρ will play the role of the time-preference parameter, γ controls risk aversion, and θ the elasticity of intertemporal substitution (EIS). Utility is defined over the composite good $C\,N^\psi$, and ψ controls the relative shares of consumption and leisure.

1.2 Solution

First, define $Z = Z_I \cdot Z_m$ and $\sigma_Z = \sqrt{\sigma_{Z,I}^2 + \sigma_{Z,m}^2}$. The Hamilton-Jacobi-Bellman equation for the social planner's optimization problem is:

$$0 \;=\; \max_{L_I, L_C, i_C, N} \left\{ h(C,N,J) + (i_C - \delta) J_{K_C} K_C + J_X X \mu_X \frac{1}{2} J_{XX} X^2 \sigma_X^2 + \mu_Z J_Z Z + \frac{1}{2} J_{ZZ} Z^2 \sigma_Z^2 \right\}$$

where

$$h(C,N,J) = \frac{\rho}{1-\theta^{-1}}\left(\frac{(CN^{\psi})^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)\,J\right)$$

subject to:

$$\begin{aligned} C &= XK_C^{\beta_C}L_C^{1-\beta_C} \\ i_C &= c^{-1}\left(\frac{X^{\alpha}Z}{K_C}K_I^{\beta_I}L_I^{1-\beta_I}\right) \\ L_C + L_I &\leq 1-N \end{aligned}$$

The labor supply decision is intratemporal. Let $N = 1 - L_C - L_I$, households will choose L_C to maximize CN^{ψ}, i.e.

$$L_C = \arg\max_{L_C} XK_C^{\beta_C}L_C^{1-\beta_C}\,(1-L_C-L_I)^{\psi} = \frac{1-\beta_C}{1-\beta_C+\psi}(1-L_I)$$

which implies that

$$N = 1 - L_C - L_I = 1 - L_I - \frac{1-\beta_C}{1-\beta_C+\psi}(1-L_I) = \frac{\psi}{1-\beta_C+\psi}(1-L_I)$$

and

$$\begin{aligned} CN^{\psi} &= XK_C^{\beta_C}(1-L_I)^{1-\beta_C+\psi}\left(\frac{1-\beta_C}{1+\psi-\beta_C}\right)^{1-\beta_C}\left(\frac{\psi}{1-\beta_C+\psi}\right)^{\psi} \\ &= XK_C^{\beta_C}(1-L_I)^{1-\beta_C+\psi}\,A_0 \\ A_0 &\equiv \left(\frac{1-\beta_C}{1+\psi-\beta_C}\right)^{1-\beta_C}\left(\frac{\psi}{1-\beta_C+\psi}\right)^{\psi} \end{aligned}$$

Replacing the above in the HJB equation along with the constraint on investment:

$$\begin{aligned} 0 &= \max_{L_I}\left\{h\left(XK_C^{\beta_C}\left(\frac{(1-\beta_C)(1-L_I)}{1+\psi-\beta_C}\right)^{1-\beta_C}, \frac{\psi}{1-\beta_C+\psi}(1-L_I), J\right) - \delta J_{K_C}K_C + \right. \\ &+ \left. c^{-1}\left(\frac{X^{\alpha}Z}{K_C}K_I^{\beta_I}L_I^{1-\beta_I}\right)J_{K_C}K_C + J_X X\mu_X + \frac{1}{2}J_{XX}X^2\sigma_X^2 + \mu_Z J_Z Z + \frac{1}{2}J_{ZZ}Z^2\sigma_Z^2\right\} \end{aligned}$$

Let $\omega = \ln\left(\frac{X^{\alpha}ZK_I^{\beta_I}}{K_C}\right)$.

$$\begin{aligned} 0 &= \max_{L_I}\left\{h\left(XK_C^{\beta_C}\left(\frac{(1-\beta_C)(1-L_I)}{1+\psi-\beta_C}\right)^{1-\beta_C}, \frac{\psi}{1-\beta_C+\psi}(1-L_I), J\right) - \delta J_{K_C}K_C + \right. \\ &+ \left. c^{-1}\left(e^{\omega}L_I^{1-\beta_I}\right)J_{K_C}K_C + J_X X\mu_X + \frac{1}{2}J_{XX}X^2\sigma_X^2 + \mu_Z J_Z Z + \frac{1}{2}J_{ZZ}Z^2\sigma_Z^2\right\} \end{aligned}$$

Guess that

$$J = \frac{(XK_C^{\beta_C})^{1-\gamma}}{1-\gamma}f(\omega)$$

Plugging in the guess, the HJB Equation becomes:

$$
\begin{aligned}
0 &= \min_{L_I} \left\{ \frac{\rho(1-\gamma)}{1-\theta^{-1}} \left(\frac{(1-L_I)^{(1-\beta_C+\psi)(1-\theta^{-1})} A_0^{1-\theta^{-1}}}{(f(\omega))^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - f(\omega) \right) - \delta \left[\beta_C (1-\gamma) f(\omega) - f'(\omega) \right] + \right. \\
&+ c^{-1} \left(e^{\omega} L_I^{1-\beta_I} \right) \left[\beta_C (1-\gamma) f(\omega) - f'(\omega) \right] + \mu_X \left[(1-\gamma) f(\omega) + \alpha f'(\omega) \right] + \\
&+ \left. \frac{1}{2}\sigma_X^2 \left[\gamma(\gamma-1) f(\omega) - \alpha(2\gamma-1) f'(\omega) + \alpha^2 f''(\omega) \right] + \mu_Z f'(\omega) + \frac{1}{2}\sigma_Z^2 \left[f''(\omega) - f'(\omega) \right] \right\} \\
0 &= \min_{L_I} \left\{ \frac{\rho(1-\gamma)}{1-\theta^{-1}} \left(\frac{(1-L_I)^{(1-\beta_C+\psi)(1-\theta^{-1})} A_0^{1-\theta^{-1}}}{(f(\omega))^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} \right) + \right. \\
&- f(\omega) \left(\frac{\rho(1-\gamma)}{1-\theta^{-1}} + \beta_C(\gamma-1)\, c^{-1} \left(e^{\omega} L_I^{1-\beta_I} \right) - \delta\, \beta_C(\gamma-1) + \mu_X (\gamma-1) - \frac{1}{2}\sigma_X^2 \gamma(\gamma-1) \right) \\
&+ f'(\omega) \left(\delta + \alpha\mu_X + \mu_Z - \alpha \left(\gamma - \frac{1}{2} \right) \sigma_X^2 - \frac{1}{2}\sigma_Z^2 - c^{-1} \left(e^{\omega} L_I^{1-\beta_I} \right) \right) \\
&+ \left. f''(\omega) \left(\frac{1}{2}\sigma_X^2 \alpha^2 + \frac{1}{2}\sigma_Z^2 \right) \right\}
\end{aligned}
$$

The first order condition with respect to L_I is

$$
A_1 (1-L_I)^{(1-\beta_C+\psi)(1-\theta^{-1})-1} L_I^{\beta_I} = \hat{i}' \left(e^{\omega} L_I^{1-\beta_I} \right) e^{\omega} \left[\beta_C + \frac{f'(\omega)}{f(\omega)} \frac{1}{\gamma-1} \right] (f(\omega))^{\frac{1-\theta^{-1}}{1-\gamma}}
$$

where $\hat{i} = c^{-1}$ and $A_1 \equiv \rho((1-\beta_C+\psi))\, A_0^{1-\theta^{-1}} (1-\beta_I)^{-1} > 0$

1.3 Prices

1.3.1 Investment goods

The relative price of investment goods is given by

$$
\xi_t = Z_m \frac{J_{K_C}}{h_{C,t}\, c'(i_C)}
$$

1.3.2 Stochastic discount factor

The IMRS is

$$
\pi_t = \exp\left(\int_0^t h_J(C_s, N_s, J_s)\, ds \right) h_C(C_t, N_t, J_t)
$$

where

$$
h_J = -\rho \frac{\gamma-\theta^{-1}}{1-\theta^{-1}} A_0^{1-\theta^{-1}} (1-L_I)^{(1-\beta_C+\psi)(1-\theta^{-1})} (f(\omega))^{-\frac{1-\theta^{-1}}{1-\gamma}} - \rho \frac{1-\gamma}{1-\theta^{-1}}
$$

and

$$
\begin{aligned}
h_C &= \rho \frac{1}{C} \frac{(CN^{\psi})^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} \\
&= \rho \frac{1}{XK_C^{\beta_C}(1-L_I)^{1-\beta_C}\left(\frac{1-\beta_C}{1+\psi-\beta_C}\right)^{1-\beta_C}} \frac{(XK_C^{\beta_C}(1-L_I)^{1-\beta_C+\psi}\,A_0)^{1-\theta^{-1}}}{((XK_C^{\beta_C})^{1-\gamma}f(\omega))^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} \\
&= (XK_C^{\beta_C})^{-\gamma} \frac{\rho\, A_0^{1-\theta^{-1}}}{\left(\frac{1-\beta_C}{1+\psi-\beta_C}\right)^{1-\beta_C}} \frac{(1-L_I)^{-\theta^{-1}(1-\beta_C)+\psi(1-\theta^{-1})}}{(f(\omega))^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} \\
&= A_2\,(XK_C^{\beta_C})^{-\gamma}\phi(\omega) \\
A_2 &\equiv \frac{\rho\, A_0^{1-\theta^{-1}}}{\left(\frac{1-\beta_C}{1+\psi-\beta_C}\right)^{1-\beta_C}} \\
\phi(\omega) &\equiv (1-l(\omega))^{-\theta^{-1}(1-\beta_C)+\psi(1-\theta^{-1})}(f(\omega))^{-\frac{\gamma-\theta^{-1}}{1-\gamma}} \\
l(\omega) &\equiv L_I
\end{aligned}
$$

The dynamics of the state variable are

$$
\begin{aligned}
d\omega &= d\ln\left(\frac{X^{\alpha}ZK_I^{\beta_I}}{K_C}\right) = \alpha d\ln X + d\ln Z - d\ln K_C \\
&= \left[\alpha(\mu_X - \frac{1}{2}\sigma_X^2) + (\mu_Z - \frac{1}{2}\sigma_Z^2) - i_C + \delta\right] dt + \alpha\sigma_X dB_t^X + \sigma_Z dB_t^Z
\end{aligned}
$$

An application of Ito's Lemma on π_t yields

$$
\begin{aligned}
d\pi_t &= \exp\left(\int_0^t h_J(C_s,N_s,J_s)\,ds\right) h_C(C_t,N_t,J_t)h_J(C_s,N_s,J_s) + \exp\left(\int_0^t h_J(C_s,N_s,J_s)\,ds\right)\mathcal{D}(h_C(C_t,N_t,J_t)) \\
\frac{d\pi_t}{\pi_t} &= h_J(C_s,N_s,J_s)\,dt + \frac{\mathcal{D}(h_C(C_t,N_t,J_t))}{h_C(C_t,N_t,J_t)}
\end{aligned}
$$

also let $\hat{\pi}(X, K_C, \omega) = h_C(C_t, N_t, J_t)$

$$
\begin{aligned}
d\hat{\pi} &= -\gamma A_2\,(XK_C^{\beta_C})^{-\gamma}\phi(\omega)\frac{dX}{X} + \frac{1}{2}\gamma(\gamma+1)A_2(XK_C^{\beta_C})^{-\gamma}\phi(\omega)\left(\frac{dX}{X}\right)^2 - \beta_C\gamma A_2\,(XK_C^{\beta_C})^{-\gamma}\phi(\omega)\frac{dK_C}{K_C} + \\
&+ A_2\,(XK_C^{\beta_C})^{-\gamma}\phi'(\omega)d\omega + \frac{1}{2}A_2\,(XK_C^{\beta_C})^{-\gamma}\phi''(\omega)(d\omega)^2 - \gamma A_2\,(XK_C^{\beta_C})^{-\gamma}\phi'(\omega)\left(\frac{dX}{X}\frac{d\omega}{\omega}\right) \\
\frac{d\hat{\pi}}{\hat{\pi}} &= -\gamma\frac{dX}{X} + \frac{1}{2}\gamma(\gamma+1)\left(\frac{dX}{X}\right)^2 - \beta_C\gamma\frac{dK_C}{K_C} + \frac{\phi'(\omega)}{\phi(\omega)}d\omega + \frac{1}{2}\frac{\phi''(\omega)}{\phi(\omega)}(d\omega)^2 - \gamma\frac{\phi'(\omega)}{\phi(\omega)}\left(\frac{dX}{X}\frac{d\omega}{\omega}\right) \\
\frac{d\hat{\pi}}{\hat{\pi}} &= -\gamma\mu_X\,dt - \gamma\sigma_X dB_t^X + \frac{1}{2}\gamma(\gamma+1)\sigma_X^2\,dt - \beta_C\gamma(i_C-\delta)\,dt + \\
&\quad+\frac{\phi'(\omega)}{\phi(\omega)}\left[\alpha(\mu_X - \frac{1}{2}\sigma_X^2) + (\mu_Z - \frac{1}{2}\sigma_Z^2) - i_C + \delta\right]dt + \frac{\phi'(\omega)}{\phi(\omega)}\left[\alpha\sigma_X dB_t^X + \sigma_Z dB_t^Z\right] \\
&\quad+\frac{1}{2}\frac{\phi''(\omega)}{\phi(\omega)}(\alpha^2\sigma_X^2 + \sigma_Z^2)\,dt - \gamma\frac{\phi'(\omega)}{\phi(\omega)}\alpha\sigma_X^2\,dt
\end{aligned}
$$

1.3.3 Investment and consumption firms

The value of the consumption firm is

$$h_{C,t}V_{C,t} = J_{K_C}K_{C,t}$$

whereas the value of the investment firms is

$$h_{C,t}V_{I,t} = \beta_I J_Z Z_{I,t}$$

Proof: Consider the value of a firm in the C-Sector. The firm buys new capital and hires labor to maximize its value

$$
\begin{aligned}
\pi_t S_t^C &= E_t\int_t^\infty \max_{L_{C,s}, i_{C,s}} \pi_s\left(X_s K_{C,s}^{\beta_C}L_{C,s}^{1-\beta_C} - w_s L_{C,s} - \xi_s c(i_C)K_C\right) \\
&= E_t\int_t^\infty \pi_s\left(\beta_C X_s K_{C,s}^{\beta_C}(L_{C,s}^*)^{1-\beta_C} - \xi_s c(i_{C,s}^*)K_{C,s}\right)\,ds \\
\Rightarrow S_t &= E_t\int_t^\infty \exp\left(\int_t^s h_J(C_u, N_u, J_u)\,du\right)\frac{h_{C,s}}{h_{C,t}}\left(\beta_C X_s K_{C,s}^{\beta_C}(L_{C,s}^*)^{1-\beta_C} - \xi_s c(i_{C,s}^*)K_{C,s}\right)\,ds
\end{aligned}
$$

The planner's Lagrangian evaluated at the optimum can be written as:

$$\mathcal{L}_t = E_t\int_t^\infty h(C_s^*, N_s^*, J_s^*) - \pi_s(C_s - X_s K_{C,s}^{\beta_C}{L_{C,s}^*}^{1-\beta_C}) - \xi_s\pi_s\left(c(i_{C,s}^*)K_{C,s} - Y_s K_I^{\beta_I}{L_I^*}^{1-\beta_I}\right)\,ds$$

The envelope theorem implies that

$$\frac{\partial\mathcal{L}}{\partial K_C} = \frac{\partial J}{\partial K_C}$$

also note that

$$\frac{\partial K_{C,s}}{\partial K_{C,t}}K_{C,t} = K_{C,s}$$

An application of the envelope theorem yields

$$\frac{\partial J_t}{\partial K_{C,t}} K_{C,t} = E_t \int_t^{\infty} \exp\left(\int_t^s h_J(C_u, N_u, J_u)\, du\right) h_{C,s} \left(\beta_C X_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - \xi_s c(i_{C,s}^*) K_{C,s}\right) ds$$

Therefore

$$S_t^C = \frac{1}{h_{C,t}} \frac{\partial J_t}{\partial K_{C,t}} K_{C,t}$$

Similarly, the value of a firm in the investment sector is:

$$\begin{aligned} \pi_t S_t^I &= E_t \int_0^{\infty} \pi_s \left(\xi_s Z_s K_I^{\beta_I} (L_{I,s})^{1-\beta_I} - w_s L_{I,s}\right) ds \\ \pi_t S_t^I &= E_t \int_0^{\infty} \pi_s \left(\xi_s \beta_I Z_s K_I^{\beta_I} (L_{I,s}^*)^{1-\beta_I}\right) ds \end{aligned}$$

Moreover, because

$$\frac{\partial J_t}{\partial Z_t} Z_t = E_t \int_t^{\infty} \exp\left(\int_t^s h_J(C_u, N_u, J_u)\, du\right) h_{C,s} \left(\xi_s Z_s K_{I,s}^{\beta_I} (L_{I,s}^*)^{1-\beta_I}\right) ds$$

this implies

$$S_t^I = \frac{1}{h_{C,t}} \beta_I J_Z Z_t$$

therefore

$$\frac{S_t^I}{S_t^C} = \frac{\beta_I f'(\omega_t)}{\beta_C (1-\gamma) f(\omega_t) - f'(\omega_t)}.$$

Also, $\frac{S^I}{S^C}$ is increasing in ω if

$$\frac{f''}{f'} > \frac{f'}{f}.$$

To see that this is the case, note that the above condition implies that $\frac{f'}{f}$ is strictly decreasing. In the regions where $f'' < 0$, the above inequality holds because the LHS is positive while the RHS is negative, so we only need to focus on the case where $f'' > 0$, where both sides are negative. Now let's consider the cases where:

1. case $\frac{f'''}{f''} > \frac{f''}{f'}$

   In this case $\frac{f''}{f'}$ is a decreasing function. Meanwhile, the slope of $\frac{f'}{f}$ depends on whether $\frac{f''}{f'} > \frac{f'}{f}$ or $\frac{f''}{f'} < \frac{f'}{f}$. We can exclude the case where

   $$\frac{f''}{f'} < \frac{f'}{f}$$

   since that would mean that $\frac{f'}{f}$, which asymptotes to 0 as $\omega \to -\infty$ is increasing with ω. Moreover, the two curves cannot cross, since if $\frac{f'}{f}$ is below $\frac{f''}{f'}$ that it is decreasing and if $\frac{f'}{f}$ is above then it is increasing. Thus the only possibility is $\frac{f''}{f'} > \frac{f'}{f}$.

2. case $\frac{f'''}{f''} < \frac{f''}{f'}$

   In this case $\frac{f''}{f'}$ is an increasing function. On the other hand, if $\frac{f''}{f'} > \frac{f'}{f}$ then $\frac{f'}{f}$ is decreasing. But we can rule out this case because $\lim_{\omega \to -\infty} \frac{f'}{f} = 0$ and the curves cannot intersect. We can also rule out the other case where $\frac{f''}{f'} > \frac{f'}{f}$, because this would imply that they are both increasing functions but by $\lim_{\omega \to -\infty} \frac{f'}{f} = 0$ and $\frac{f'}{f} \in (1-\gamma, 0)$ this is not possible.

From the above, the only possibility then is that $\frac{f''}{f'} > \frac{f'}{f}$.

1.3.4 Value of assets in place and growth opportunities

Consider the value of a firm in the C-sector that plans to not invest in the future

$$\pi_t S^V_{C,t} = E_t \int_t^\infty \pi_s \left(X_s (K_{C,t} e^{-\delta\, s})^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} \right) ds$$

its labor decision yields

$$(1-\beta_C) X_s (K_{C,0} e^{-\delta\, s})^{\beta_C} L_{C,s}^{1-\beta_C} = w_s L_{C,s}$$

Let $\Lambda_{s,t} = \exp(\int_t^s i^*_{C,u}\, du)$. Now consider a firm who follows the optimal investment policy, its first order condition is

$$(1-\beta_C) X_s K_{C,s}{}^{\beta_C} L^*_{C,s}{}^{1-\beta_C} = w_s L^*_{C,s}$$

dividing through, this yields

$$L_{C,s} = L^*_{C,s} \Lambda^{-1}_{s,t}$$

which implies

$$\begin{aligned}
\pi_t S^V_t &= E_t \int_t^\infty \pi_s X_s\, K_{C,t}^{\beta_C} e^{-\delta \beta_C\, s} \hat{L}_{C,0}^{1-\beta_C}\, ds \\
\pi_t S^V_t &= E_t \int_t^\infty \pi_s \beta_C X_s\, K_{C,s}^{\beta_C} L_{C,t}^{1-\beta_C} \Lambda^{-1}_{s,t}\, ds \\
h_{C,t} S^V_t &= E_t \int_t^\infty \exp\left(\int_t^s h_J(C,N,J) - i_{C,u}\, du \right) \beta_C h_{C,s} C ds \\
&= \frac{X_t^{1-\gamma} K_t^{\beta_C(1-\gamma)}}{1-\gamma} \times \\
&\quad Et \int_t^\infty \exp\left(\int_t^s \hat{\rho}_u du \right) \rho \beta_C (1-\gamma) L_{C,s}^{(1-\beta_C)(1-\theta^{-1})} N_s^{\psi(1-\theta^{-1})} f(\omega_s)^{\frac{\gamma-\theta^{-1}}{\gamma-1}}\, ds \\
&= \frac{X_t^{1-\gamma} K_{C,t}^{\beta_C(1-\gamma)}}{1-\gamma} g(\omega_t)
\end{aligned}$$

The Feynman-Kac theorem implies that $g(\omega_t)$ can be computed as the solution to the ODE:

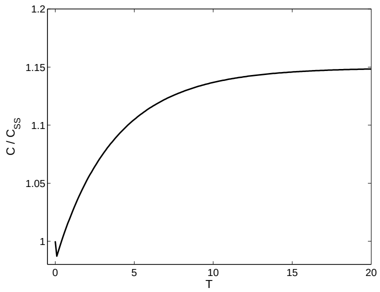
$$0 = \rho \beta_C (1-\gamma) L_C(\omega)^{(1-\beta_C)(1-\theta^{-1})} N(\omega)^{\psi(1-\theta^{-1})} f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} + \hat{\rho}(\omega) g(\omega) + \mathcal{D}_\omega g(\omega)$$

where

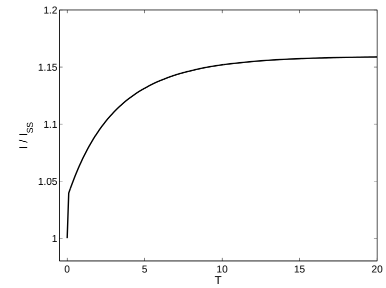
$$\hat{\rho}(\omega_u) \;=\; h_J(\omega) + (\beta_C(1-\gamma)-1) i_{C,u} - \delta\beta_C(1-\gamma) + (1-\gamma)\left(\mu_X - \frac{1}{2}\sigma_X^2\right) + \frac{1}{2}(1-\gamma)^2 \sigma_X^2.$$

1.4 Figures

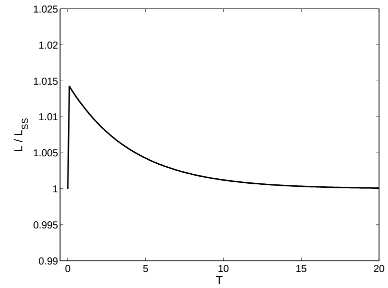
Figure 1: Dynamic responses to the Investment-Specific shock (Z)



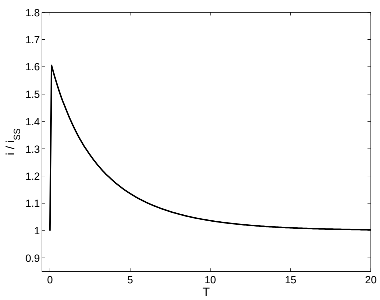
(a) Consumption (C)



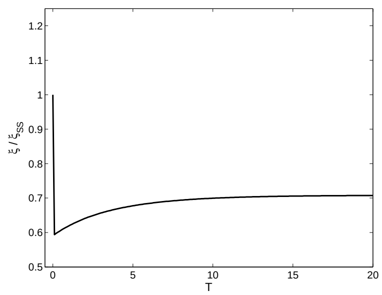
(b) Investment (I)



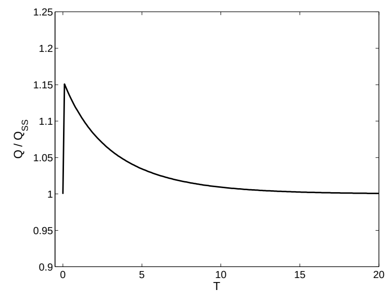
(c) Labor Supply ($L_C + L_I$)



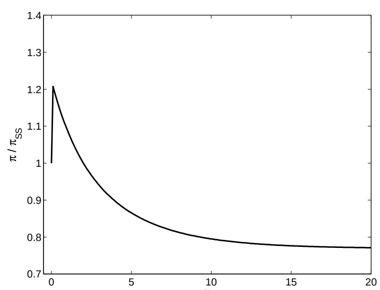
(d) Investment rate (i_C)



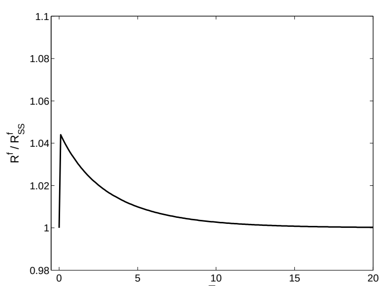
(e) Price of Capital Goods / MEI (ξ/Z_2)



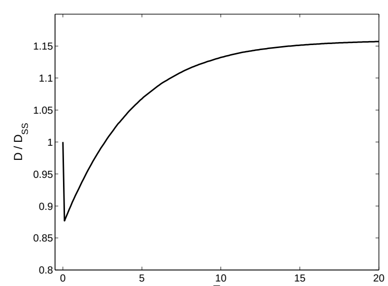
(f) Tobin's Q



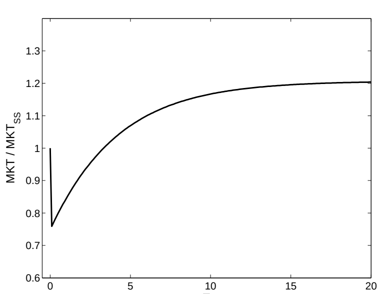
(g) State Price Density (π)



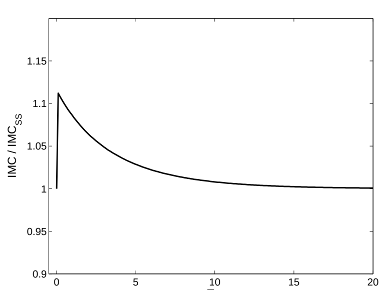
(h) Risk-free rate (r_f)



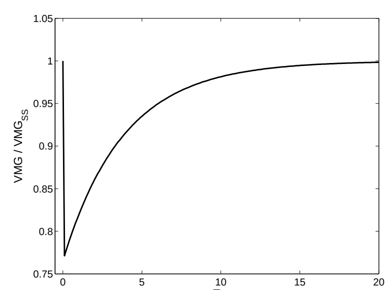
(i) Aggregate Dividend



(j) Market Portfolio: Price

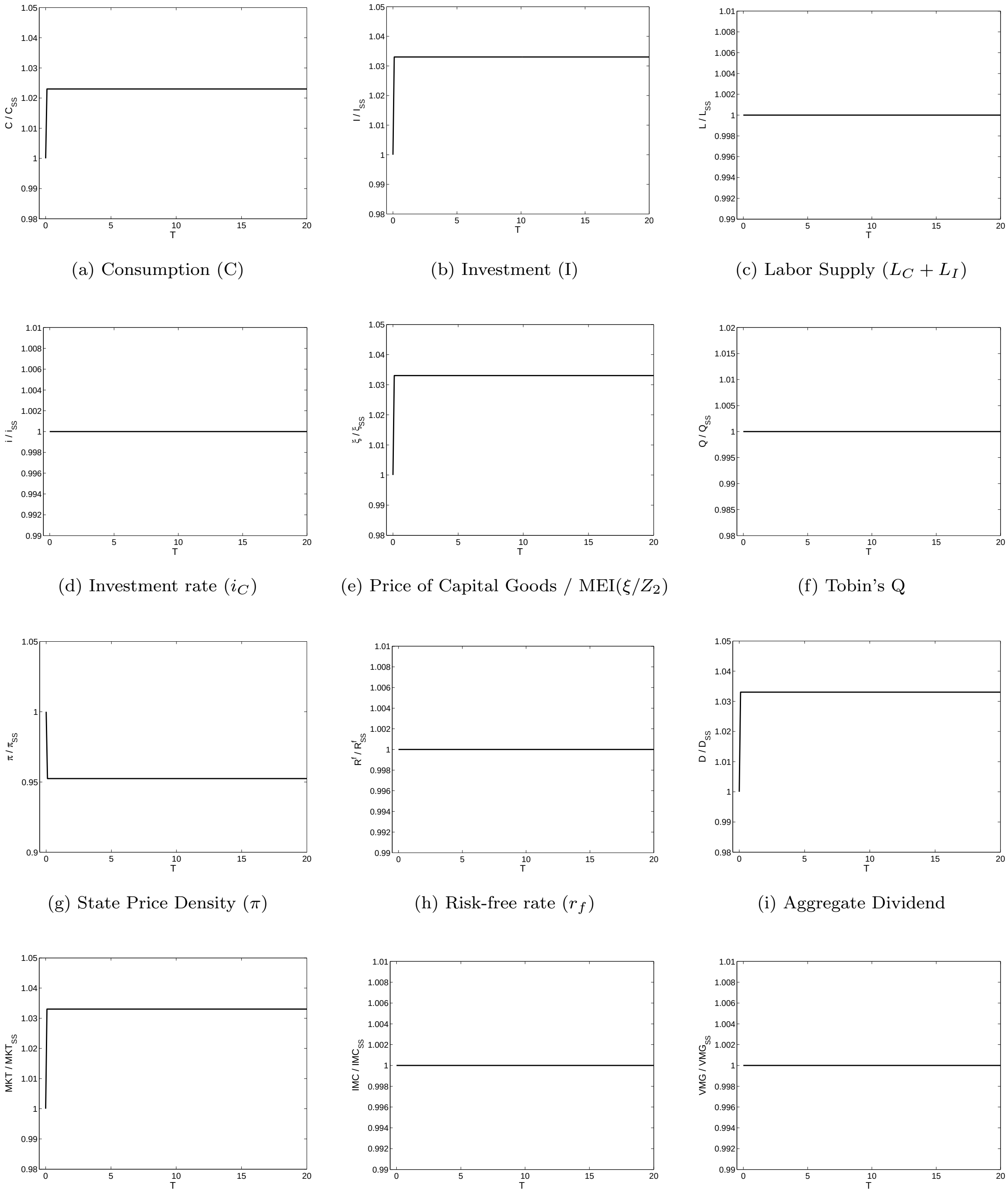


(k) Value of Investment firms relative to Consumption firms



(l) Value of Assets in place relative to Growth Opportunities

Figure 2: Dynamic responses to the Consumption shock (X)



(a) Consumption (C)

(b) Investment (I)

(c) Labor Supply ($L_C + L_I$)

(d) Investment rate (i_C)

(e) Price of Capital Goods / MEI(ξ/Z_2)

(f) Tobin's Q

(g) State Price Density (π)

(h) Risk-free rate (r_f)

(i) Aggregate Dividend

(j) Market Portfolio: Price

(k) Value of Investment firms relative to Consumption firms

(l) Value of Assets in place relative to Growth Opportunities

1.5 Calibration and comparative statics

Table 1: Parameters used for benchmark calibration

Parameter	Symbol	Value
Preferences		
Discount rate	ρ	0.001
Elasticity of intertemporal Substitution	θ	0.3
Relative risk aversion	γ	1.1
Leisure share in utility	ψ	3
Technology		
Growth rate of C-shock	μ_X	0.1%
Volatility of C-shock	σ_X	2.0%
Growth rate of I-shock (I-TFP)	μ_Z	4.0%
Volatility of I-shock (I-TFP)	$\sigma_{Z,1}$	3.5%
Volatility of I-shock (MEI)	$\sigma_{Z,2}$	13.5%
Sensitivity of Y_I to C-shock	α	0
Production		
Capital share in C-sector	β_C	0.3
Capital share in I-sector	β_I	0.1
Adjustment cost parameter	λ	1.15
Depreciation rate of capital	δ	8.5%

Table 2: Model versus Data: Macroeconomic Quantities

	$\mu(\%)$	$\sigma(\%)$	ρ	Correlation			
A: Model							
				$\dot{c}$	$\dot{i}$	$\dot{l}$	$\dot{y}$
$\dot{c}$	0.99	1.86	0.37				
	[-0.04, 2.02]	[1.50, 2.30]	[0.12, 0.60]				
$\dot{i}$	1.00	3.82	0.22	0.42			
	[-0.13, 2.08]	[3.05, 4.84]	[-0.01, 0.43]	[0.17, 0.61]			
$\dot{l}$	-0.00	0.78	0.53	-0.09	0.87		
	[-0.08, 0.08]	[0.63, 0.96]	[0.34, 0.68]	[-0.31, 0.14]	[0.78, 0.93]		
$\dot{y}$	0.99	1.92	0.36	0.94	0.71	0.27	
	[-0.04, 2.02]	[1.56, 2.37]	[0.10, 0.58]	[0.89, 0.96]	[0.56, 0.81]	[0.04, 0.48]	
$\dot{\xi}$	-2.95	3.27	0.25	0.55	0.18	-0.10	0.50
	[-4.13, -1.76]	[2.69, 3.90]	[0.01, 0.46]	[0.34, 0.71]	[-0.07, 0.42]	[-0.33, 0.16]	[0.28, 0.67]
B: Data							
	$\mu(\%)$	$\sigma(\%)$	ρ	Correlation			
				$\dot{c}$	$\dot{i}$	$\dot{l}$	$\dot{y}$
$\dot{c}$	2.51	1.95	0.40				
	[2.09, 2.93]	[1.65, 2.24]	[0.21, 0.58]				
$\dot{i}$	2.60	6.22	0.17	0.39			
	[1.26, 3.94]	[5.26, 7.18]	[-0.04, 0.38]	[0.21, 0.58]			
$\dot{l}$	-0.08	2.52	0.16	0.41	0.83		
	[-0.62, 0.47]	[2.13, 2.13]	[-0.06, 0.37]	[0.23, 0.59]	[0.76, 0.90]		
$\dot{y}$	2.35	3.24	0.10	0.84	0.67	0.64	
	[1.66, 3.05]	[2.74, 3.74]	[-0.12, 0.32]	[0.78, 0.90]	[0.55, 0.79]	[0.51, 0.76]	
$\dot{\xi}$	-3.78	3.01	0.18	0.44	-0.06	-0.26	0.24
	[-4.42, -3.13]	[2.55, 3.47]	[-0.03, 0.39]	[0.26, 0.61]	[-0.28, 0.16]	[-0.46, -0.05]	[0.03, 0.44]

Table 2 compares moments of the data to simulated moments from the model. The first three columns report means (μ), standard deviations (σ) and first-order autocorrelations (ρ). The last five columns report correlations. The top panel shows moments from simulated data. I simulate 50,000 samples, each with a length of 100 years. I drop the first half of the sample to remove the impact of initial values. I simulate the model at a monthly frequency (dt=1/12) and then aggregate the data to form annual observations. $\dot{y}$ refers to log output growth, $\dot{c}$ refers to log consumption growth, $\dot{i}$ to log investment growth, $\dot{l}$ to log growth of labor supply and $\dot{\xi}$ to log growth of investment goods prices. I report median moments along with the 5% and 95% percentiles across simulations. The bottom panel shows moments in actual data. I use annual data in the 1951:2008 period. Output is GDP excluding government consumption, consumption is non-durables plus services, labor supply is non-farm business hours, investment is non-residential fixed investment and the relative price of investment comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I deflate quantities by population and nominal variables by the consumption deflator.

Table 3: Simulated Moments: Asset Prices

	Model			Data
	median	5%	95%	
$E[R_M - r_f]$	1.31	-1.62	4.16	4.89
$\sigma[R_M - r_f]$	12.33	11.74	12.93	17.92
$E[R_I - r_f]$	1.04	-1.34	3.38	3.75
$\sigma[R_I - r_f]$	10.16	9.67	10.66	18.96
$E[R_C - r_f]$	1.44	-1.78	4.51	5.17
$\sigma[R_C - r_f]$	13.60	12.95	14.26	14.77
$E[R_I - R_C]$	-0.39	-1.16	0.45	-1.41
$\sigma[R_I - R_C]$	3.48	3.28	3.67	10.96
$E[r_f]$	2.83	-0.65	6.36	2.90
$\sigma[R_f]$	3.61	2.42	5.36	3.00
$E[R_V - R_G]$	1.12	0.99	1.24	-
$\sigma[R_V - R_G]$	10.96	9.99	11.76	-

Table 3 compares moments of the data to simulated moments from the model. The left three columns shows moments from simulated data. I simulate 10,000 samples, each with a length of 100 years. I drop the first half of the sample to remove the impact of initial values. I simulate the model at a monthly frequency (dt=1/12) and then aggregate the data to form annual observations. R_M refers to returns of the market portfolio, R_I to returns on the investment sector, R_C to returns on the consumption sector, R_V to returns on a pure value firm in the consumption sector, R_G to returns on a pure growth firm in the consumption sector, and R_f to the risk-free rate. In simulations, I compute risk premia by $E[r_i - r_f] = \frac{1}{T}\sum[r_{it} - r_{ft}]$, except for the risk premium on the pure value factor, which I compute using $E[R_V - R_G] = -cov(\frac{d\pi}{\pi}, R_{Vt} - R_{Gt})$. I report median values along with 5% and 95% percentiles across simulations. The fourth column shows the corresponding moments in the data. The mean and volatility of the market portfolio are computed using data from Kenneth French's website over the period 1962:2008. The moments for investment and consumption firms are computed over the 1962:2008 period. I classify firms as investment and consumption producers based on NIPA Tables and NAICS codes, see online appendix for details. The mean of the real risk-free rate are from the long sample of Campbell and Cochrane (1999). The volatility of the interest rate is from Chan and Kogan (2002) and it refers to the volatility of the ex-post real rate.

Table 4: COMPARATIVE STATICS WITH RESPECT TO α

	Model				Data
	BENCH	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 2$	
$E[R_M - r_f]$	1.29	1.26	1.25	1.25	4.89
$\sigma[R_M - r_f]$	12.33	12.22	12.17	12.26	17.92
$E[R_I - R_C]$	-0.36	-0.35	-0.36	-0.37	-1.41
$\sigma[R_I - R_C]$	3.48	3.50	3.54	3.68	10.96
$E[r_f]$	2.84	2.89	2.97	3.04	2.90
$\sigma[R_f]$	3.60	3.62	3.67	3.80	3.00
$E[R_V - R_G]$	1.12	1.11	1.11	1.14	-
$\sigma[R_V - R_G]$	10.96	11.01	11.12	11.46	-
$\sigma[\dot{c}]$	1.86	1.86	1.87	1.89	1.94
$\sigma[\dot{i}]$	3.82	3.96	4.09	4.43	6.22
$\sigma[\dot{\xi}]$	3.27	2.96	2.84	3.19	3.07
$\rho[\dot{c}, \dot{i}]$	0.42	0.46	0.50	0.56	0.39
$\rho[\dot{y}, \dot{i}]$	0.71	0.74	0.76	0.80	0.67
$\rho[\dot{c}, \dot{y}]$	0.94	0.94	0.94	0.94	0.84
$\rho[\dot{c}, \dot{l}]$	-0.08	-0.03	0.03	0.14	0.41

Table **??** compares moments of the data to simulated moments from the model using three different calibrated values for the correlation between the output of the investment and consumption sector (λ). The left four columns shows moments from simulated data. I simulate 10,000 samples, each with a length of 100 years. I throw out the first half of the sample to remove the impact of initial values. I simulate the model at a monthly frequency (dt=1/12) and then aggregate the data to form annual observations. R_M refers to returns of the market portfolio, R_I to returns on the investment sector, R_C to returns on the consumption sector, R_V to returns on a pure value firm in the consumption sector, R_V to returns on a pure growth firm in the consumption sector, and R_f to the risk-free rate. $\dot{y}$ refers to log output growth, $\dot{c}$ refers to log consumption growth, $\dot{i}$ to log investment growth, and $\dot{\xi}$ to log growth of investment goods prices. In simulations, I compute risk premia as $E[r_i - r_f] = -cov(\frac{d\pi}{\pi}, r_{it})$. I report median values along with 5% and 95% percentiles across simulations. The fourth column shows the corresponding moments in the data. The mean and volatility of the market portfolio are computed using data from Kenneth French's website over the period 1962:2008. The moments for investment and consumption firms are computed over the 1962:2008 period. I classify firms as investment and consumption producers based on NIPA Tables and NAICS codes. The mean of the real risk-free rate are from the long sample of Campbell and Cochrane (1999). The volatility of the interest rate is from Chan and Kogan (2002) and it refers to the volatility of the ex-post real rate. Output is GDP excluding government consumption, consumption is non-durables plus services, labor supply is non-farm business hours, investment is non-residential fixed investment and the relative price of investment comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I deflate quantities by population and nominal variables by the consumption deflator.

2 A Model with flexible capital in the I-sector

This model allows the capital stock in the investment sector to vary.

2.1 Setup

The consumption sector uses Capital (K_C) and Labor (L_C) to produce the consumption good according to the following technology:

$$Y_C = X K_C^{\beta_C} L_C^{1-\beta_C},$$

where

$$dX_t = \mu_X \, X_t \, dt + \sigma_X X_t dB_t^x.$$

The investment sector also uses capital and labor (K_I) and (L_I) to produce the investment good

$$Y_I = Z_I K_I^{\beta_I} L_I^{1-\beta_I},$$

where

$$dZ_{I,t} = \mu_z \, Z_{I,t} \, dt + \sigma_{1,Z} Z_{I,t} dB_t^{1,z}$$

Investing in either entails some costs of adjustment. Increasing the capital stock by I costs a total of $Z_m{}^{-}1 \, c(I/K)K$ units of the investment good, where

$$c(i) = \frac{1}{\lambda}(1+i)^{\lambda} - \frac{1}{\lambda}$$

the function $c(i)$ is the cost of investment plus the adjustment costs. It satisfies $c(0) = 0$ and $c'(0) = 1$, and

$$dZ_{m,t} = \sigma_{Z,2} Z_{m,t} dB_t^{1,z}.$$

Households supply $1 - N_t$ units of labor that can be freely allocated between the two sectors,

$$L_{C,t} + L_{I,t} = 1 - N_t. \tag{3}$$

Households have preference over consumption, and leisure according to:

$$h(C, N, J) = \frac{\rho}{1-\theta^{-1}} \left(\frac{(CN^{\psi})^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)\, J \right) \tag{4}$$

Here ρ will play the role of the time-preference parameter, γ controls risk aversion, and θ the elasticity of intertemporal substitution (EIS). Utility is defined over the composite good $C \, N^{\psi}$, and ψ controls the relative shares of consumption and leisure.

2.2 Solution

Define $Z = Z_I \times Z_m$. The Hamilton-Jacobi-Bellman equation for the social planner's optimization problem is:

$$\begin{aligned}
0 \quad = \quad & \max_{L_I, L_C, i_C, i_I, N} \Big\{ h(C,N,J) + (i_C - \delta) J_{K_C} K_C + (i_I - \delta) J_{K_I} K_I + J_X X \mu_X + \frac{1}{2} J_{XX} X^2 \sigma_X^2 \\
& + \mu_z Z J_Z + \frac{1}{2} J_{ZZ} Z^2 \sigma_Z^2 \Big\}
\end{aligned}$$

where

$$h(C,N,J) = \frac{\rho}{1-\theta^{-1}} \left(\frac{(CN^{\psi})^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma) J \right)$$

subject to:

$$\begin{aligned}
C &= X K_C^{\beta_C} L_C^{1-\beta_C} \\
c(i_C) K_C + c(i_I) K_I &= X^{\alpha} Z K_I^{\beta_I} L_I^{1-\beta_I} \\
\Rightarrow i_C &= c^{-1} \left(\frac{X^{\alpha} Z}{K_C} K_I^{\beta_I} L_I^{1-\beta_I} - c(i_I) \frac{K_I}{K_C} \right) \\
L_C + L_I &\leq 1 - N
\end{aligned}$$

The labor supply decision is intratemporal. Let $N = 1 - L_C - L_I$, households will choose L_C to maximize CN^{ψ}, i.e.

$$L_C = \arg\max_{L_C} X K_C^{\beta_C} L_C^{1-\beta_C} (1 - L_C - L_I)^{\psi} = \frac{1-\beta_C}{1-\beta_C+\psi}(1-L_I)$$

which implies that

$$N = 1 - L_C - L_I = 1 - L_I - \frac{1-\beta_C}{1-\beta_C+\psi}(1-L_I) = \frac{\psi}{1-\beta_C+\psi}(1-L_I)$$

and

$$\begin{aligned}
CN^{\psi} &= X K_C^{\beta_C} (1-L_I)^{1-\beta_C+\psi} \left(\frac{1-\beta_C}{1+\psi-\beta_C} \right)^{1-\beta_C} \left(\frac{\psi}{1-\beta_C+\psi} \right)^{\psi} \\
&= X K_C^{\beta_C} (1-L_I)^{1-\beta_C+\psi} A_0 \\
A_0 &\equiv \left(\frac{1-\beta_C}{1+\psi-\beta_C} \right)^{1-\beta_C} \left(\frac{\psi}{1-\beta_C+\psi} \right)^{\psi}
\end{aligned}$$

Replacing the above in the HJB equation along with the constraint on investment:

$$\begin{aligned}
0 \quad = \quad & \max_{L_I, i_I} \Bigg\{ h\left(X K_C^{\beta_C} \left(\frac{(1-\beta_C)(1-L_I)}{1+\psi-\beta_C} \right)^{1-\beta_C}, \frac{\psi}{1-\beta_C+\psi}(1-L_I), J \right) - \delta J_{K_C} K_C + \\
+ \quad & c^{-1} \left(\frac{Z}{K_C} K_I^{\beta_I} L_I^{1-\beta_I} - c(i_I) \frac{K_I}{K_C} \right) J_{K_C} K_C + (i_I - \delta) K_I J_{K_I} \\
& + J_X X \mu_X + \frac{1}{2} J_{XX} X^2 \sigma_X^2 + \mu_z J_Z Z + \frac{1}{2} J_{ZZ} Z^2 \sigma_Z^2 \Bigg\}
\end{aligned}$$

Let $\omega = \ln\left(\frac{ZK_I^{\beta_I}}{K_C}\right)$, $k = \ln\left(\frac{K_I}{K_C}\right)$.

$$
\begin{aligned}
0 &= \max_{L_I, i_I} \Bigg\{ h\left(XK_C^{\beta_C} \left(\frac{(1-\beta_C)(1-L_I)}{1+\psi-\beta_C} \right)^{1-\beta_C}, \frac{\psi}{1-\beta_C+\psi}(1-L_I), J \right) - \delta J_{K_C} K_C + \\
&+ c^{-1}\left(e^{\omega} L_I^{1-\beta_I} - c(i_I)e^{k}\right) J_{K_C} K_C + (i_I - \delta) J_{K_I} K_I + J_X X \mu_X + \frac{1}{2} J_{XX} X^2 \sigma_X^2 + \mu_z J_Z Z + \frac{1}{2} J_{ZZ} Z^2 \sigma_Z^2 \Bigg\}
\end{aligned}
$$

Guess that

$$
J = \frac{(XK_C^{\beta_C})^{1-\gamma}}{1-\gamma} f(\omega, k)
$$

and plugging in the guess, the HJB Equation becomes:

$$
\begin{aligned}
0 &= \min_{L_I, i_I} \Bigg\{ \frac{\rho(1-\gamma)}{1-\theta^{-1}} \left(\frac{(1-L_I)^{(1-\beta_C+\psi)(1-\theta^{-1})} \, A_0^{1-\theta^{-1}}}{(f(\omega))^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} \right) + \\
&- f(\omega,k)\left(\frac{\rho(1-\gamma)}{1-\theta^{-1}} + \beta_C(\gamma-1)\, c^{-1}\left(e^{\omega} L_I^{1-\beta_I} - c(i_I)e^{k}\right) - \delta\, \beta_C(\gamma-1) + \mu_X\,(\gamma-1) - \frac{1}{2}\sigma_X^2 \gamma(\gamma-1) \right) \\
&+ f_1(\omega,k)\left(\delta + \mu_Z + \beta_I(i_I - \delta) - \frac{1}{2}\sigma_Z^2 - c^{-1}\left(e^{\omega} L_I^{1-\beta_I} - c(i_I)e^{k}\right) \right) \\
&+ f_{11}(\omega,k)\left(\frac{1}{2}\beta^2 \sigma_Z^2 \right) \\
&+ f_2(\omega,k)\left(i_I - c^{-1}\left(e^{\omega} L_I^{1-\beta_I} - c(i_I)e^{k}\right) \right)
\end{aligned}
$$

The first order condition with respect to L_I and i_I are

$$
A_1\,(1-L_I)^{(1-\beta_C+\psi)(1-\theta^{-1})-1}\, L_I^{\beta_I} = \hat{i}'\left(e^{\omega} L_I^{1-\beta_I} - c(i_I)\,e^{k}\right) e^{\omega} \left[\beta_C + \frac{f_1(\omega,k) + f_2(\omega,k)}{(\gamma-1) f(\omega,k)} \right] (f(\omega,k))^{\frac{1-\theta^{-1}}{1-\gamma}}
$$

$$
[\beta_I f_1(\omega,k) + f_2(\omega,k)] = \hat{i}'\left(e^{\omega} L_I^{1-\beta_I} - c(i_I)e^{k}\right) c'(i_I) e^{k} \left[\beta_C(1-\gamma)\, f(\omega,k) - f_1(\omega,k) - f_2(\omega,k) \right]
$$

where $\hat{i} = c^{-1}$ and $A_1 \equiv \rho((1-\beta_C+\psi))\, A_0^{1-\theta^{-1}} (1-\beta_I)^{-1} > 0$

2.3 Prices

2.3.1 Investment goods

The relative price of investment goods is given by

$$
\xi_t = \frac{J_{K_C}}{h_{C,t}\, c'(i_C)} =
$$

given the first order condition for investment, it also equals

$$
\xi_t = \frac{J_{K_I}}{h_{C,t}\, c'(i_I)}
$$

as long as $i_I > 0$.

2.3.2 Stochastic discount factor

The state-price density is

$$\pi_t = \exp\left(\int_0^t h_J(C_s, N_s, J_s)\, ds\right) h_C(C_t, N_t, J_t)$$

where

$$h_J \quad = \quad -\rho \frac{\gamma - \theta^{-1}}{1-\theta^{-1}} A_0^{1-\theta^{-1}} (1-L_I)^{(1-\beta_C+\psi)(1-\theta^{-1})} (f(\omega))^{-\frac{1-\theta^{-1}}{1-\gamma}} - \rho \frac{1-\gamma}{1-\theta^{-1}}$$

$$\begin{aligned}
h_C &= A_2\, (X K_C^{\beta_C})^{-\gamma} \phi(\omega) \\
A_2 &\equiv \frac{\rho\, A_0^{1-\theta^{-1}}}{\left(\frac{1-\beta_C}{1+\psi-\beta_C}\right)^{1-\beta_C}} \\
\phi(\omega) &\equiv (1-l(\omega))^{-\theta^{-1}(1-\beta_C)+\psi(1-\theta^{-1})} (f(\omega))^{-\frac{\gamma-\theta^{-1}}{1-\gamma}} \\
l(\omega) &\equiv L_I
\end{aligned}$$

$$\begin{aligned}
d\pi_t &= \exp\left(\int_0^t h_J(C_s, N_s, J_s)\, ds\right) h_C(C_t, N_t, J_t) h_J(C_s, N_s, J_s) + \exp\left(\int_0^t h_J(C_s, N_s, J_s)\, ds\right) \mathcal{D}(h_C(C_t, N_t, J_t)) \\
\frac{d\pi_t}{\pi_t} &= h_J(C_s, N_s, J_s)\, dt + \frac{\mathcal{D}(h_C(C_t, N_t, J_t))}{h_C(C_t, N_t, J_t)}
\end{aligned}$$

2.3.3 Investment and consumption firms

The value of the consumption firm is

$$\pi_t V_{C,t} = J_{K_C} K_{C,t}$$

whereas the value of the investment firms is

$$\pi_t V_{I,t} = J_{K_I} K_{I,t}$$

Proof: Consider the value of a firm in the C-Sector. The firm buys new capital and hires labor to maximize its value

$$\begin{aligned}
\pi_t S_t^C &= E_t \int_t^\infty \max_{L_{C,s}, i_{C,s}} \pi_s \left(X_s K_{C,s}^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} - \xi_s c(i_C) K_C\right) \\
&= E_t \int_t^\infty \pi_s \left(\beta_C X_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - \xi_s c(i_{C,s}^*) K_{C,s}\right) ds \\
\Rightarrow S_t &= E_t \int_t^\infty \exp\left(\int_t^s h_J(C_u, N_u, J_u)\, du\right) \frac{h_{C,s}}{h_{C,t}} \left(\beta_C X_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - \xi_s c(i_{C,s}^*) K_{C,s}\right) ds
\end{aligned}$$

The planner's Lagrangian evaluated at the optimum can be written as:

$$\mathcal{L}_t = E_t \int_t^\infty h(C_s^*, N_s^*, J_s^*) - \pi_s (C_s - X_s K_{C,s}^{\beta_C} {L_{C,s}^*}^{1-\beta_C}) - \xi_s \pi_s \left(c(i_{C,s}^*) K_{C,s} - Y_s K_I^{\beta_I} {L_I^*}^{1-\beta_I} \right) ds$$

The envelope theorem implies that

$$\frac{\partial \mathcal{L}}{\partial K_C} = \frac{\partial J}{\partial K_C}$$

also note that

$$\frac{\partial K_{C,s}}{\partial K_{C,t}} K_{C,t} = K_{C,s}$$

An application of the envelope theorem yields

$$\frac{\partial J_t}{\partial K_{C,t}} K_{C,t} = E_t \int_t^\infty \exp\left(\int_t^s h_J(C_u, N_u, J_u)\, du \right) h_{C,s} \left(\beta_C X_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - \xi_s c(i_{C,s}^*) K_{C,s} \right) ds$$

Therefore

$$S_t^C = \frac{1}{h_{C,t}} \frac{\partial J_t}{\partial K_{C,t}} K_{C,t}$$

2.3.4 Value of assets in place and growth opportunities

Consider the value of a firm in the C-sector that plans to not invest in the future

$$\pi_0 S_{C,0}^V = E_0 \int_0^\infty \pi_s \left(X_s (K_{C,0} e^{-\delta\, s})^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} \right) ds$$

its labor decision yields

$$(1-\beta_C) X_s (K_{C,0} e^{-\delta\, s})^{\beta_C} L_{C,s}^{1-\beta_C} = w_s L_{C,s}$$

Let $\Lambda_{s,t} = \exp(\int_t^s i_{C,u}^*\, du)$. Now consider a firm who follows the optimal investment policy, its first order condition is

$$(1-\beta_C) X_s {K_{C,s}}^{\beta_C} {L_{C,s}^*}^{1-\beta_C} = w_s L_{C,s}^*$$

dividing through, this yields

$$L_{C,s} = L_{C,s}^* \Lambda_{s,0}^{-1}$$

which implies

$$\begin{aligned} S_{C,t}^V &= E_t \int_t^\infty \frac{\pi_s}{\pi_t} X_s\, K_{C,0}^{\beta_C} e^{-\delta \beta_C\, s} \hat{L}_{C,0}^{1-\beta_C}\, ds \\ h_{C,t} S_{C,t}^V &= \frac{X_t^{1-\gamma} K_{C,t}^{\beta_C(1-\gamma)}}{1-\gamma} g(\omega_t, k_t) \end{aligned}$$

The Feynman-Kac theorem implies that $g(\omega_t, k_t)$ can be computed as the solution to the PDE:

$$0 = \rho \beta_C (1-\gamma) L_C(\omega,k)^{(1-\beta_C)(1-\theta^{-1})} N(\omega,k)^{\psi(1-\theta^{-1})} f(\omega,k)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} + \hat{\rho}(\omega,k) g(\omega,k) + \mathcal{D}_{\omega,k} g(\omega,k)$$

where

$$\hat{\rho}(\omega_u, k_u) = h_J(C_u, N_u, J_u) + (\beta_C(1-\gamma) - 1)i_{C,u} - \delta\beta_C(1-\gamma) + (1-\gamma)\left(\mu_X - \frac{1}{2}\sigma_X^2\right) + \frac{1}{2}(1-\gamma)^2\sigma_X^2.$$

The value of assets in place in the investment sector is

$$\begin{aligned} S_{I,t}^V &= \max_{\hat{L}_I} E_t \int_t^{\infty} \frac{\pi_s}{\pi_t} \left(Z_s (K_{I,t} e^{-\delta(s-t)})^{\beta_I} \hat{L}_I^{1-\beta_I} - w_s \hat{L}_{I,s} \right) ds \\ h_{C,t} S_{I,t}^V &= \frac{X_t^{1-\gamma} K_{C,t}^{\beta_C(1-\gamma)}}{1-\gamma} j(\omega_t, k_t) \end{aligned}$$

where $j(\omega_t, k_t)$ can be computed as the solution to the PDE:

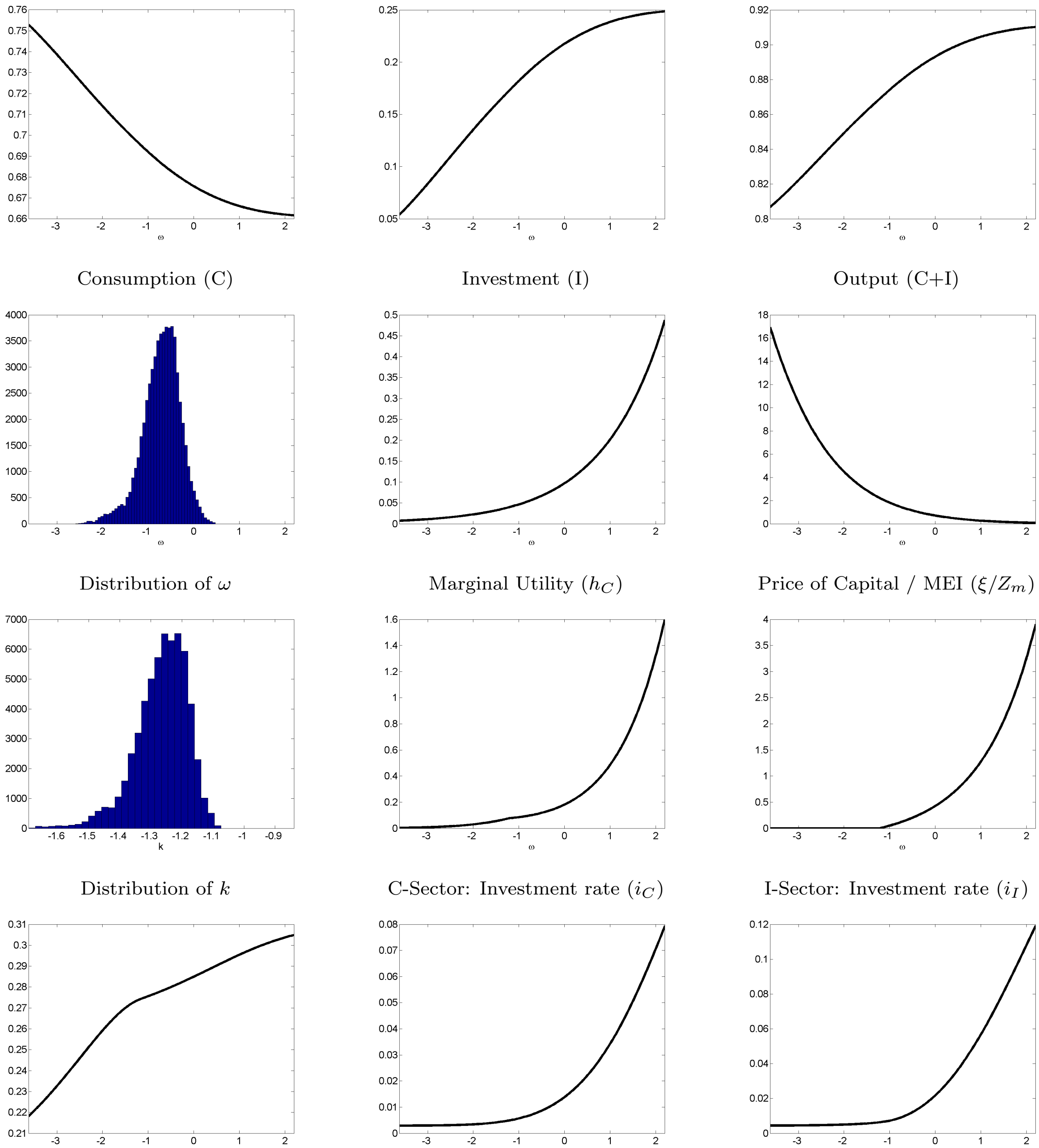
$$\beta_I \frac{1}{c'(i_C)} \left[\beta_C(1-\gamma) f(\omega, k) - f_1(\omega, k) - f_2(\omega, k)\right] e^{\omega_s} L_I(\omega_s, k_s)^{1-\beta_I} + \hat{\rho}_I(\omega_u, k_u) j(\omega, k) + \mathcal{D}_{\omega,k}\, j(\omega, k)$$

where

$$\hat{\rho}_I(\omega_u, k_u) = h_J(C_u, N_u, J_u) + (\beta_C(1-\gamma))i_{C,u} - i_{I,u} - \delta\beta_C(1-\gamma) + (1-\gamma)\left(\mu_X - \frac{1}{2}\sigma_X^2\right) + \frac{1}{2}(1-\gamma)^2\sigma_X^2.$$

2.4 Figures

Figure 3: Model Solution



Consumption (C) | Investment (I) | Output (C+I)

Distribution of ω | Marginal Utility (h_C) | Price of Capital / MEI (ξ/Z_m)

Distribution of k | C-Sector: Investment rate (i_C) | I-Sector: Investment rate (i_I)

Value of investment firms relative to consumption firms | C-Sector: value of growth relative to assets in place | I-Sector: value of growth relative to assets in place

Table 3 plots the numerical solution of the model. I evaluate the above aggregate quantities and prices at $K_C = 1$, $X = 1$ and $k = \bar{k}$, where $\bar{k}$ is the mean of the stationary distribution of k. I plot them as a function of ω. I obtain the joint distribution of ω and k by simulating one long path from the model of 10,000 years. I drop the first half to remove the dependence on initial values.

2.5 Calibration

Table 5: Parameters used for calibration

Parameter	Symbol	Value
Preferences		
Discount rate	ρ	0.001
Elasticity of intertemporal Substitution	θ	0.3
Relative risk aversion	γ	1.1
Leisure share in utility	ψ	3
Technology		
Growth rate of C-shock	μ_X	0.1%
Volatility of C-shock	σ_X	2.0%
Growth rate of I-shock (I-TFP)	μ_Z	4.0%
Volatility of I-shock (I-TFP)	$\sigma_{Z,1}$	3.5%
Volatility of I-shock (MEI)	$\sigma_{Z,2}$	13.5%
Production		
Capital share in C-sector	β_C	0.3
Capital share in I-sector	β_I	0.3
Adjustment cost parameter	λ	1.15
Depreciation rate of capital	δ	8.5%

Table 6: Model versus Data: Macroeconomic Quantities

	$\mu(\%)$	$\sigma(\%)$	ρ	Correlation			
A: Model							
				$\dot{c}$	$\dot{i}$	$\dot{l}$	$\dot{y}$
$\dot{c}$	1.27	1.98	0.42				
	[-0.17, 2.75]	[1.56, 2.54]	[0.14, 0.67]				
$\dot{i}$	1.27	3.23	0.29	0.42			
	[-0.26, 2.79]	[2.54, 4.21]	[0.04, 0.51]	[0.10, 0.65]			
$\dot{l}$	-0.00	0.20	0.56	-0.22	0.80		
	[-0.03, 0.03]	[0.16, 0.25]	[0.38, 0.70]	[-0.44, 0.01]	[0.64, 0.90]		
$\dot{y}$	1.27	1.94	0.42	0.95	0.69	0.11	
	[-0.17, 2.75]	[1.53, 2.51]	[0.13, 0.67]	[0.90, 0.97]	[0.50, 0.82]	[-0.12, 0.33]	
$\dot{\xi}$	-3.82	3.90	0.23	0.33	0.08	-0.13	0.29
	[-4.99, -2.65]	[3.22, 4.64]	[-0.00, 0.44]	[0.05, 0.55]	[-0.17, 0.34]	[-0.36, 0.14]	[0.01, 0.52]
B: Data							
	$\mu(\%)$	$\sigma(\%)$	ρ	Correlation			
				$\dot{c}$	$\dot{i}$	$\dot{l}$	$\dot{y}$
$\dot{c}$	2.51	1.95	0.40				
	[2.09, 2.93]	[1.65, 2.24]	[0.21, 0.58]				
$\dot{i}$	2.60	6.22	0.17	0.39			
	[1.26, 3.94]	[5.26, 7.18]	[-0.04, 0.38]	[0.21, 0.58]			
$\dot{l}$	-0.08	2.52	0.16	0.41	0.83		
	[-0.62, 0.47]	[2.13, 2.13]	[-0.06, 0.37]	[0.23, 0.59]	[0.76, 0.90]		
$\dot{y}$	2.35	3.24	0.10	0.84	0.67	0.64	
	[1.66, 3.05]	[2.74, 3.74]	[-0.12, 0.32]	[0.78, 0.90]	[0.55, 0.79]	[0.51, 0.76]	
$\dot{\xi}$	-3.78	3.01	0.18	0.44	-0.06	-0.26	0.24
	[-4.42, -3.13]	[2.55, 3.47]	[-0.03, 0.39]	[0.26, 0.61]	[-0.28, 0.16]	[-0.46, -0.05]	[0.03, 0.44]

Table 6 compares moments of the data to simulated moments from the model. The first three columns report means (μ), standard deviations (σ) and first-order autocorrelations (ρ). The last five columns report correlations. The top panel shows moments from simulated data. I simulate 10,000 samples, each with a length of 100 years. I drop the first half of the sample to remove the impact of initial values. I simulate the model at a monthly frequency (dt=1/12) and then aggregate the data to form annual observations. $\dot{y}$ refers to log output growth, $\dot{c}$ refers to log consumption growth, $\dot{i}$ to log investment growth, $\dot{l}$ to log growth of labor supply and $\dot{\xi}$ to log growth of investment goods prices. I report median moments along with the 5% and 95% percentiles across simulations. The bottom panel shows moments in actual data. I use annual data in the 1951:2008 period. Output is GDP excluding government consumption, consumption is non-durables plus services, labor supply is non-farm business hours, investment is non-residential fixed investment and the relative price of investment comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I deflate quantities by population and nominal variables by the consumption deflator.

Table 7: Simulated Moments: Asset Prices

	Model			Data
	median	5%	95%	
$E[R_M - r_f]$	1.72	1.56	1.88	4.89
$\sigma[R_M - r_f]$	14.54	13.83	15.25	17.92
$E[R_I - R_C]$	-0.05	-0.06	-0.05	-1.41
$\sigma[R_I - R_C]$	0.76	0.64	0.90	10.96
$E[r_f]$	4.49	-0.37	9.53	2.90
$\sigma[R_f]$	4.37	3.11	6.65	3.00
$E[S_I/S_C]$	28.54	25.59	30.44	17.85
$E[I/C]$	28.14	24.87	30.20	19.27
$E[R_{C,V} - R_{C,G}]$	1.51	1.18	1.72	-
$\sigma[R_{C,V} - R_{C,G}]$	13.15	10.69	14.44	-
$E[R_{I,V} - R_{I,G}]$	1.81	1.34	2.12	-
$\sigma[R_{I,V} - R_{I,G}]$	15.97	12.48	17.89	-

Table 7 compares moments of the data to simulated moments from the model. The left three columns shows moments from simulated data. I simulate 10,000 samples, each with a length of 100 years. I drop the first half of the sample to remove the impact of initial values. I simulate the model at a monthly frequency (dt=1/12) and then aggregate the data to form annual observations. R_M refers to returns of the market portfolio, R_I to returns on the investment sector, R_C to returns on the consumption sector, $R_{i,V}$ to returns on a pure value firm in sector $i \in \{I, C\}$, $R_{i,G}$ to returns on a pure growth firm in the consumption sector, and R_f to the risk-free rate. In simulations, I compute risk premia as $E[r_i - r_f] = -cov(\frac{d\pi}{\pi}, r_{it})$. I report median values along with 5% and 95% percentiles across simulations. The fourth column shows the corresponding moments in the data. The mean and volatility of the market portfolio are computed using data from Kenneth French's website over the period 1962:2008. The moments for investment and consumption firms are computed over the 1962:2008 period. I classify firms as investment and consumption producers based on NIPA Tables and NAICS codes. The mean of the real risk-free rate are from the long sample of Campbell and Cochrane (1999). The volatility of the interest rate is from Chan and Kogan (2002) and it refers to the volatility of the ex-post real rate.

3 A model with Jaimovitch-Rebelo preferences

This model features preferences as in Jaimovich and Rebelo (2009).

3.1 Setup

The consumption sector uses Capital (K_C) and Labor (L_C) to produce the consumption good according to the following technology:

$$Y_C = X K_C^{\beta_C} L_C^{1-\beta_C},$$

where

$$dX_t = \mu_X \, X_t \, dt + \sigma_X X_t dB_t^x.$$

The investment sector also uses capital and labor (K_I) and (L_I) to produce the investment good

$$Y_I = Z_I K_I^{\beta_I} L_I^{1-\beta_I},$$

where

$$dZ_{I,t} = \mu_z \, Z_{I,t} \, dt + \sigma_{1,Z} Z_{I,t} dB_t^{1,z}$$

Investing in in the C-sector entails some costs of adjustment. Increasing the capital stock by I costs a total of $Z_m{}^{-}1 \, c(I/K)K$ units of the investment good, where

$$c(i) = \frac{1}{\lambda}(1+i)^{\lambda} - \frac{1}{\lambda}$$

the function $c(i)$ is the cost of investment plus the adjustment costs. It satisfies $c(0) = 0$ and $c'(0) = 1$, and

$$dZ_{m,t} = \sigma_{Z,2} Z_{m,t} dB_t^{1,z}$$

Households supply L_t units of labor that can be freely allocated between the two sectors,

$$L_{C,t} + L_{I,t} = L_t. \tag{5}$$

Households have preference over consumption, and labor according to:

$$h(C, L, \tilde{x}, J) = \frac{\rho}{1-\theta^{-1}} \left(\frac{(C - \chi \, L^{\psi} \, C^{\kappa} \, e^{(1-\kappa)\tilde{x}})^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma) \, J \right) \tag{6}$$

where

$$d\tilde{x}_t = \kappa(\ln C_t - \tilde{x}_t) \, dt$$

which can be written as

$$\tilde{x}_t = \tilde{x}_0 + \kappa \int_0^t e^{-\kappa} \ln C_s \, ds$$

Here, preferences are as in Jaimovich and Rebelo (2009). Utility is defined over the composite good $C - \chi \, L^{\psi} \, C^{\kappa} \, e^{(1-\kappa)\,\tilde{x}}$, and χ controls the relative shares of consumption and leisure, ψ controls the elasticity of labor supply, and the parameter κ controls the income effect on labor. Varying κ between 0 and 1, changes preferences from Greenwood, Hercowitz and Huffman (1988) to King, Plosser and Rebelo (1988). Here ρ will play the role of the time-preference parameter, γ controls risk aversion, and θ the elasticity of intertemporal substitution (EIS) over the composite good and *not* consumption.

3.2 Solution

Define $Z = Z_I \times Z_m$. The Hamilton-Jacobi-Bellman equation for the social planner's optimization problem is:

$$
\begin{aligned}
0 \quad = \quad & \max_{L,L_I,L_C,i_C} \Big\{ h(C,N,\tilde{x},J) + J_{K_C} K_C \left(i_C - \delta\right) + J_X X \mu_X \frac{1}{2} J_{XX} X^2 \sigma_X^2 + \mu_Z J_Z Z + \frac{1}{2} J_{ZZ} Z^2 \sigma_Z^2 \\
& + J_{\tilde{x}} \kappa (\ln C - \tilde{x}) \}
\end{aligned}
$$

where

$$
h(C,L,\tilde{x},J) = \frac{\rho}{1-\theta^{-1}} \left(\frac{(C - \chi \, L^{\psi} \, C^{\kappa} \, e^{(1-\kappa)\tilde{x}})^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)\, J \right)
$$

subject to:

$$
\begin{aligned}
C &= X K_C^{\beta_C} L_C^{1-\beta_C} \\
i_C &= c^{-1} \left(\frac{Z}{K_C} K_I^{\beta_I} L_I^{1-\beta_I} \right) \\
L_C + L_I &\leq L
\end{aligned}
$$

Let $\omega = \ln\left(\frac{X^{\alpha} Z K_I^{\beta_I}}{K_C}\right)$ and $x = \tilde{x} - \ln X - \beta_C \ln K$. Guess that

$$
J = \frac{(X K_C^{\beta_C})^{1-\gamma}}{1-\gamma} f(\omega, x)
$$

Replacing the above in the HJB equation along with the constraint on investment:

$$
\begin{aligned}
0 \quad = \quad & \max_{L_I,L_C} \left\{ \frac{\rho(1-\gamma)}{1-\theta^{-1}} \frac{(L_C^{1-\beta_C} - \chi \,(L_C + L_I)^{\psi} \, L_C^{\kappa(1-\beta_C)} \, e^{(1-\kappa)\,h})^{1-\theta^{-1}}}{f(\omega,x)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} \right. \\
& - f(\omega,x) \left[\frac{\rho(1-\gamma)}{1-\theta^{-1}} - \beta_C (1-\gamma) \left(c^{-1}\left(e^{\omega} L_I^{1-\beta_I} \right) - \delta \right) - \mu_X \,(1-\gamma) - \frac{1}{2}\sigma_X^2 \,\gamma\,(\gamma-1) \right] \\
& + f_1(\omega,x) \left[\mu_Z + \delta - \frac{1}{2}\sigma_Z^2 - c^{-1}\left(e^{\omega} L_I^{1-\beta_I} \right) \right] \\
& + f_{11}(\omega,x) \left[\frac{1}{2}\sigma_Z^2 \right] \\
& + f_2(\omega,x) \left[\kappa\left((1-\beta_C)\ln L_C - h\right) + \frac{1}{2}\sigma_X^2 (2\gamma - 1) - \mu_X - \beta_C \left(c^{-1}\left(e^{\omega} L_I^{1-\beta_I} \right) - \delta \right) \right] \\
& + f_{22}(\omega,x) \left[\frac{1}{2}\sigma_X^2 \right]
\end{aligned}
$$

3.3 Prices

The State-Price Density is given by (see Duffie and Skiadas (1994)):

$$\pi_t = \exp\left(\int_0^t h_J(C_s, L_s, \tilde{x}, J_s)\, ds\right) \left\{ h_C(C_t, L_t, \tilde{x}, J_t) + \kappa \frac{1}{C_t} E_t \left[\int_t^\infty \exp\left(\int_t^s h_J(C_s, L_s, \tilde{x}_s, J_s)\, ds + \kappa(s-t) \right) h_{\tilde{x}} \right] \right\}$$

To compute it:

$$\begin{aligned}
h_J &= -\rho \left({L_C}^{1-b_C} - \chi\, L^\psi {L_C}^{\kappa(1-b_C)} e^{(1-\kappa)x} \right)^{1-\theta^{-1}} \frac{\gamma - \theta^{-1}}{1-\theta^{-1}} f(\omega,x)^{\frac{1-\theta^{-1}}{\gamma-1}} - \rho \frac{1-\gamma}{1-\theta^{-1}} \\
h_C &= (X {K_C}^{\beta_C})^{-\gamma} \rho \left({L_C}^{1-b_C} - \chi\, L^\psi {L_C}^{\kappa(1-\beta_C)} e^{(1-\kappa)x} \right)^{-\theta^{-1}} \left(1 - \chi\, L^\psi {L_C}^{-(1-\kappa)(1-\beta_C)} \kappa\, e^{(1-\kappa)x} \right) \times f(\omega,x)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \\
h_{\tilde{x}} &= -(X {K_C}^{\beta_C})^{1-\gamma} \rho \left({L_C}^{1-b_C} - \chi\, L^\psi {L_C}^{\kappa(1-b_C)} e^{(1-\kappa)x} \right)^{-\theta^{-1}} \chi\, L^\psi {L_C}^{\kappa(1-b_C)} (1-\kappa)\, e^{(1-\kappa)x} f(\omega,x)^{\frac{\gamma-\theta^{-1}}{\gamma-1}}
\end{aligned}$$

We can write the state price density as

$$\pi_t = B_t \tilde{\pi}_t$$

where B_t is the discounting and $\tilde{\pi}_t$ is the conversion from utils into dollars. Each component equals:

$$\begin{aligned}
B_t &= \exp\left(-\rho \int_0^t \left[\left({L_{C,s}}^{1-b_C} - \chi\, {L_s}^\psi {L_{C,s}}^{\kappa(1-b_C)} e^{(1-\kappa)x_s} \right)^{1-\theta^{-1}} \frac{\gamma-\theta^{-1}}{1-\theta^{-1}} f(\omega_s, x_s)^{\frac{1-\theta^{-1}}{\gamma-1}} + \frac{1-\gamma}{1-\theta^{-1}} \right] ds \right) \\
\tilde{\pi}_t &= (X {K_C}^{\beta_C})^{-\gamma} \left[\Pi^0(\omega_t, x_t) + \kappa L_{C,t}^{\beta_C - 1}\, \Pi^1(\omega_t, x_t) \right]
\end{aligned}$$

where the two functions $\Pi^0(\omega_t, x_t)$ and $\Pi^1(\omega_t, x_t)$ satisfy

$$\begin{aligned}
\Pi^0(\omega,x) &= \left({L_C}^{1-b_C} - \chi\, L^\psi {L_C}^{\kappa(1-\beta_C)} e^{(1-\kappa)x} \right)^{-\theta^{-1}} \left(1 - \chi\, L^\psi {L_C}^{-(1-\kappa)(1-\beta_C)} \kappa\, e^{(1-\kappa)x} \right) \times f(\omega,x)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \\
-\mathcal{D}_{\omega,x}\left[\Pi^1(\omega,x)\right] &= \rho \left({L_C}^{1-b_C} - \chi\, L^\psi {L_C}^{\kappa(1-b_C)} e^{(1-\kappa)x} \right)^{-\theta^{-1}} \chi\, L^\psi {L_C}^{\kappa(1-b_C)} (1-\kappa)\, e^{(1-\kappa)x} f(\omega,x)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \\
&\quad - \left(\kappa - h_J - \beta_C(1-\gamma) \left(c^{-1} \left(e^\omega L_I^{1-\beta_I} \right) - \delta \right) - \mu_X (1-\gamma) - \frac{1}{2}\sigma_X^2\, \gamma(\gamma-1) \right) \Pi^1(\omega,x)
\end{aligned}$$

The two state variables ω and x, under certain parameter restrictions have stationary distributions. Their dynamics are given by:

$$\begin{aligned}
d\omega &= d\ln Z - d\ln K_C \\
&= \left[(\mu_Z - \frac{1}{2}\sigma_Z^2) - i_C(\omega,x) + \delta \right] dt + \sigma_Z dB_t^Z \\
dx &= d\tilde{x} - d\,\ln X - \beta_C d\ln K_C \\
&= \left[\mu_X - \frac{1}{2}\sigma_X^2 + \beta_C \delta - \beta_C i_C(\omega,x) + (1-\beta_C)\ln L_C(\omega,x) - x \right] dt - \sigma_X dB_t^X
\end{aligned}$$

The relative price of investment goods is given by

$$\xi_t = Z_m \frac{J_{K_C}}{\tilde{\pi}_t\, c'(i_C)}$$

The value of the consumption firm is

$$\tilde{\pi}_t V_{C,t} = J_{K_C} K_{C,t}$$

whereas the value of the investment firms is

$$\tilde{\pi}_t V_{I,t} = \beta_I J_Z Z_{I,t}$$

The value of assets in place in the consumption sector can be computed as before based on

$$\begin{aligned}
\pi_t S_t^V =& E_t \int_t^{\infty} \pi_s X_s\, K_{C,t}^{\beta_C} e^{-\delta \beta_C\, s} \hat{L}_{C,0}^{1-\beta_C}\, ds && (7)\\
\pi_t S_t^V =& E_t \int_t^{\infty} \pi_s \beta_C X_s\, K_{C,s}^{\beta_C} L_{C,t}^{1-\beta_C} \Lambda_{s,t}^{-1}\, ds && (8)\\
\tilde{\pi}_t S_t^V =& E_t \int_t^{\infty} \exp\left(\int_t^s h_J(C,N,J) - i_{C,u}\, du \right) \beta_C \tilde{\pi}_t C ds && (9)\\
=& E_t \int_t^{\infty} \exp\left(\int_t^s h_J(C,N,J) - i_{C,u}\, du \right) \beta_C (X_s {K_{C,s}}^{\beta_C})^{1-\gamma} \left[\Pi^0(\omega_s, x_s) L_{C,s}^{1-\beta_C} + \kappa\, \Pi^1(\omega_s, x_s) \right] ds && (10)\\
=& (X_t K_{C,t}^{\beta_C})^{1-\gamma} E_t \int_t^{\infty} \exp\left(\int_t^s \hat{\rho}_u\, du \right) \beta_C \left[\Pi^0(\omega_s, x_s) L_{C,s}^{1-\beta_C} + \kappa\, \Pi^1(\omega_s, x_s) \right] ds && (11)\\
=& (X_t K_{C,t}^{\beta_C})^{1-\gamma} h(\omega_t, x_t) && (12)
\end{aligned}$$

where $h(\omega_t, x_t)$ satisfies the following PDE:

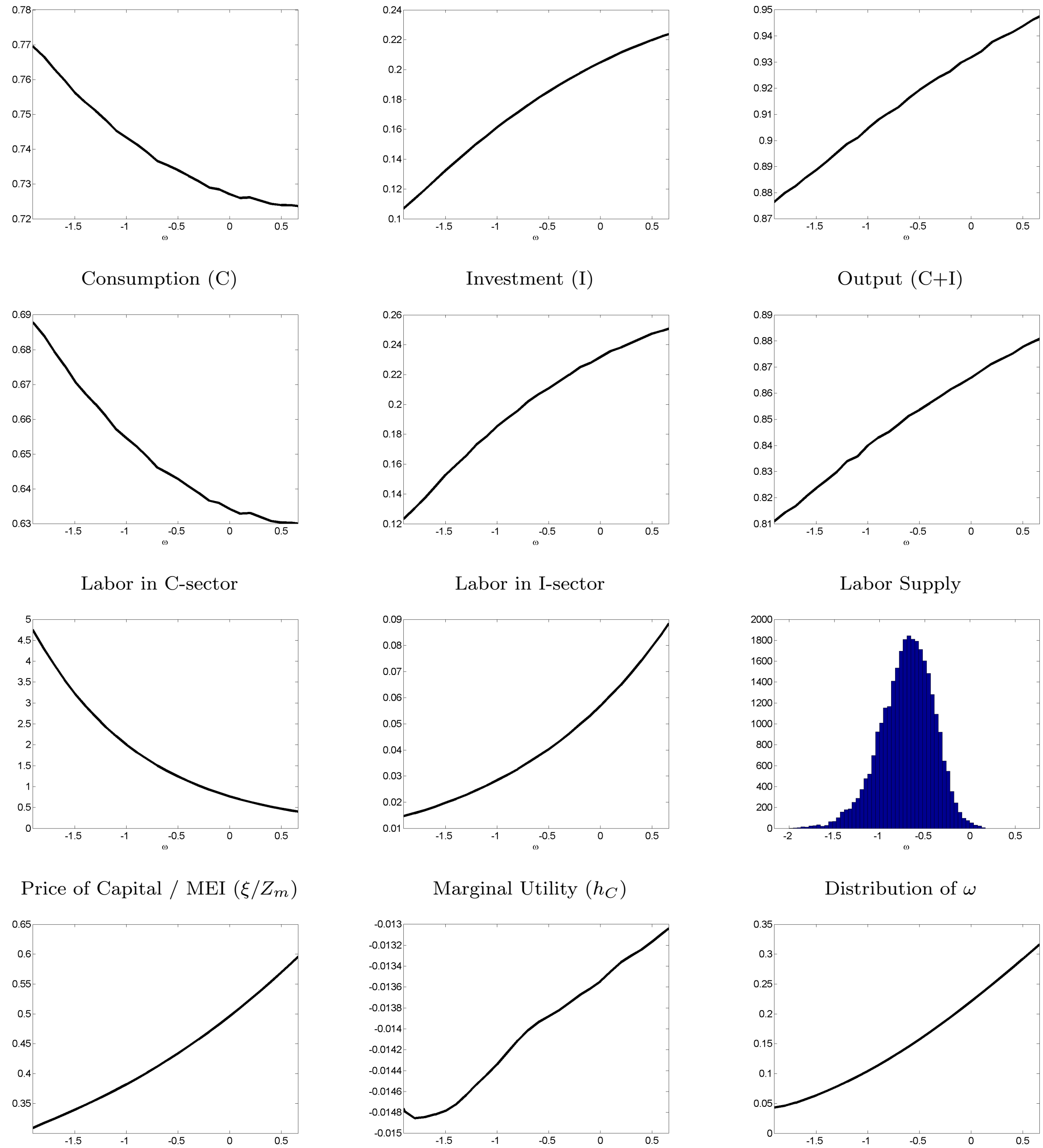
$$0 = \beta_C \left[\Pi^0(\omega, x) L_C(\omega, x)^{1-\beta_C} + \kappa\, \Pi^1(\omega, x) \right] + \hat{\rho}(\omega, x) h(\omega, x) + \mathcal{D}_{\omega,x} h(\omega, x)$$

and

$$\hat{\rho}(\omega, x)) = h_J(\omega, x) + (\beta_C(1-\gamma) - 1) i_C(\omega, x) - \delta \beta_C (1-\gamma) + (1-\gamma) \left(\mu_X - \frac{1}{2} \sigma_X^2 \right) + \frac{1}{2} (1-\gamma)^2 \sigma_X^2$$

3.4 Figures

Figure 4: Model Solution as function of $\omega = \ln Z - \ln K_C$



Consumption (C) | Investment (I) | Output (C+I)

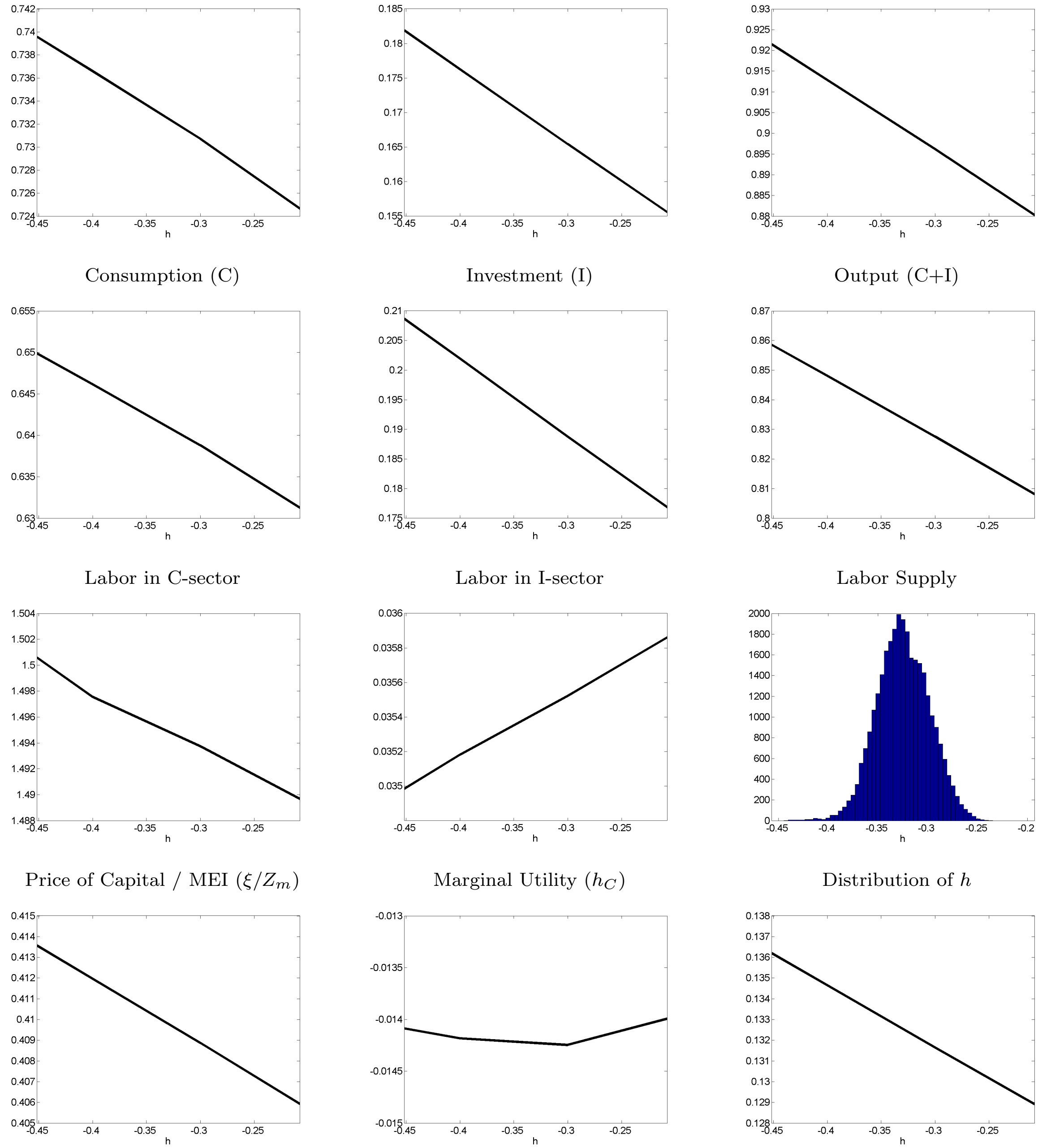
Labor in C-sector | Labor in I-sector | Labor Supply

Price of Capital / MEI (ξ/Z_m) | Marginal Utility (h_C) | Distribution of ω

Value of investment firms relative to consumption firms | Risk premium for Z shock (b_Z) | Value of growth opportunities relative to assets in place

Table 4 plots the numerical solution of the model. I evaluate the above aggregate quantities and prices at $K_C = 1$, $X = 1$ and $h = \bar{k}$, where $\bar{h}$ is the mean of the stationary distribution of h. I plot them as a function of ω. I obtain the joint distribution of ω and h by simulating one long path from the model of 10,000 years. I drop the first half to remove the dependence on initial values.

Figure 5: Model Solution as function of $h = \tilde{h} - \ln X - \beta_C \ln K_C$



Consumption (C) | Investment (I) | Output (C+I)

Labor in C-sector | Labor in I-sector | Labor Supply

Price of Capital / MEI (ξ/Z_m) | Marginal Utility (h_C) | Distribution of h

Value of investment firms relative to consumption firms | Risk premium for Z shock (b_Z) | Value of growth opportunities relative to assets in place

Table 5 plots the numerical solution of the model. I evaluate the above aggregate quantities and prices at $K_C = 1$, $X = 1$ and $\omega = \bar{\omega}$, where $\bar{\omega}$ is the mean of the stationary distribution of ω. I plot them as a function of h. I obtain the joint distribution of ω and h by simulating one long path from the model of 10,000 years. I drop the first half to remove the dependence on initial values.

3.5 Calibration

Table 8: Parameters used for calibration

Parameter	Symbol	Value
Preferences		
Discount rate	ρ	0.001
Elasticity of intertemporal Substitution	θ	0.3
Relative risk aversion	γ	1.1
Share of leisure in utility	χ	0.5
Elasticity of Labor Supply	ψ	1.5
Strength of Short-Run Wealth Effect on Labor	κ	0.75
Technology		
Growth rate of C-shock	μ_X	0.1%
Volatility of C-shock	σ_X	2.0%
Growth rate of I-shock (I-TFP)	μ_Z	4.0%
Volatility of I-shock (I-TFP)	$\sigma_{Z,1}$	3.5%
Volatility of I-shock (MEI)	$\sigma_{Z,2}$	13.5%
Production		
Capital elasticity in C-sector	β_C	0.3
Capital elasticity in I-sector	β_I	0.3
Adjustment cost parameter	λ	1.15
Depreciation rate of capital	δ	8.5%

Table 9: Model versus Data: Macroeconomic Quantities

	$\mu(\%)$	$\sigma(\%)$	ρ	Correlation			
A: Model							
				$\dot{c}$	$\dot{i}$	$\dot{l}$	$\dot{y}$
$\dot{c}$	1.23	2.05	0.37				
	[0.11, 2.34]	[1.65, 2.55]	[0.12, 0.60]				
$\dot{i}$	1.23	4.45	0.22	0.45			
	[-0.01, 2.42]	[3.55, 5.65]	[-0.01, 0.43]	[0.21, 0.63]			
$\dot{l}$	-0.00	0.73	0.55	0.28	0.94		
	[-0.07, 0.07]	[0.60, 0.87]	[0.37, 0.69]	[0.08, 0.46]	[0.90, 0.97]		
$\dot{y}$	1.23	2.17	0.34	0.94	0.73	0.58	
	[0.11, 2.35]	[1.75, 2.67]	[0.09, 0.57]	[0.90, 0.97]	[0.59, 0.82]	[0.42, 0.71]	
$\dot{\xi}$	-3.71	3.33	0.27	0.57	0.29	0.19	0.55
	[-4.96, -2.44]	[2.74, 3.99]	[0.02, 0.48]	[0.36, 0.72]	[0.04, 0.51]	[-0.04, 0.41]	[0.34, 0.71]
B: Data							
	$\mu(\%)$	$\sigma(\%)$	ρ	Correlation			
				$\dot{c}$	$\dot{i}$	$\dot{l}$	$\dot{y}$
$\dot{c}$	2.51	1.95	0.40				
	[2.09, 2.93]	[1.65, 2.24]	[0.21, 0.58]				
$\dot{i}$	2.60	6.22	0.17	0.39			
	[1.26, 3.94]	[5.26, 7.18]	[-0.04, 0.38]	[0.21, 0.58]			
$\dot{l}$	-0.08	2.52	0.16	0.41	0.83		
	[-0.62, 0.47]	[2.13, 2.13]	[-0.06, 0.37]	[0.23, 0.59]	[0.76, 0.90]		
$\dot{y}$	2.35	3.24	0.10	0.84	0.67	0.64	
	[1.66, 3.05]	[2.74, 3.74]	[-0.12, 0.32]	[0.78, 0.90]	[0.55, 0.79]	[0.51, 0.76]	
$\dot{\xi}$	-3.78	3.01	0.18	0.44	-0.06	-0.26	0.24
	[-4.42, -3.13]	[2.55, 3.47]	[-0.03, 0.39]	[0.26, 0.61]	[-0.28, 0.16]	[-0.46, -0.05]	[0.03, 0.44]

Table 9 compares moments of the data to simulated moments from the model. The first three columns report means (μ), standard deviations (σ) and first-order autocorrelations (ρ). The last five columns report correlations. The top panel shows moments from simulated data. I simulate 10,000 samples, each with a length of 100 years. I drop the first half of the sample to remove the impact of initial values. I simulate the model at a monthly frequency (dt=1/12) and then aggregate the data to form annual observations. $\dot{y}$ refers to log output growth, $\dot{c}$ refers to log consumption growth, $\dot{i}$ to log investment growth, $\dot{l}$ to log growth of labor supply and $\dot{\xi}$ to log growth of investment goods prices. I report median moments along with the 5% and 95% percentiles across simulations. The bottom panel shows moments in actual data. I use annual data in the 1951:2008 period. Output is GDP excluding government consumption, consumption is non-durables plus services, labor supply is non-farm business hours, investment is non-residential fixed investment and the relative price of investment comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I deflate quantities by population and nominal variables by the consumption deflator.

Table 10: Simulated Moments: Asset Prices

	Model			Data
	median	5%	95%	
$E[R_M - r_f]$	1.53	1.38	1.68	4.46
$\sigma[R_M - r_f]$	13.62	12.96	14.29	18.40
$E[R_I - R_C]$	-0.43	-0.47	-0.38	-1.41
$\sigma[R_I - R_C]$	3.93	3.71	4.14	10.96
$E[r_f]$	4.13	1.35	6.83	2.90
$\sigma[R_f]$	4.94	3.87	6.40	3.00
$E[R_V - R_G]$	1.33	1.58	1.16	-
$\sigma[R_V - R_G]$	12.37	11.15	13.79	-

Table 10 compares moments of the data to simulated moments from the model. The left three columns shows moments from simulated data. I simulate 10,000 samples, each with a length of 100 years. I drop the first half of the sample to remove the impact of initial values. I simulate the model at a monthly frequency (dt=1/12) and then aggregate the data to form annual observations. R_M refers to returns of the market portfolio, R_I to returns on the investment sector, R_C to returns on the consumption sector, $R_{i,V}$ to returns on a pure value firm in sector $i \in \{I, C\}$, $R_{i,G}$ to returns on a pure growth firm in the consumption sector, and R_f to the risk-free rate. In simulations, I compute risk premia as $E[r_i - r_f] = -cov(\frac{d\pi}{\pi}, r_{it})$. I report median values along with 5% and 95% percentiles across simulations. The fourth column shows the corresponding moments in the data. The mean and volatility of the market portfolio are computed using data from Kenneth French's website over the period 1962:2008. The moments for investment and consumption firms are computed over the 1962:2008 period. I classify firms as investment and consumption producers based on NIPA Tables and NAICS codes. The mean of the real risk-free rate are from the long sample of Campbell and Cochrane (1999). The volatility of the interest rate is from Chan and Kogan (2002) and it refers to the volatility of the ex-post real rate.

4 Numerical Solution

I solve the Hamilton-Jacobi-Bellman equation directly using numerical methods. See Kushner and Dupuis (1992) for a textbook treatment on numerical solutions of stochastic optimal control problems in continuous time.

For exposition purposes consider benchmark model and focus on the case where $\gamma = \theta^{-1}$ and $\psi = 0$. In this case the HJB equation becomes:

$$0 = \min_l \left\{ (1-l)^{(1-\gamma)(1-\beta_C)} - (u + \beta_C(\gamma-1)c^{-1}(e^{\omega} l^{1-\beta_I}))f(\omega) + f'(\omega)(\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega} l^{1-\beta_I})) + \frac{1}{2}\sigma_Y^2 f''(\omega) \right\}$$

where u is a constant and ω follows

$$d\omega = (\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega} l^{*1-\beta_I}))\, dt + \sigma_Y dZ_t^Y$$

I discretize the state space, creating a grid for ω and f with $h = \Delta\omega$. Then the following approximations can be used

$$f'(\omega_n) \approx \frac{f_{n+1} - f_{n-1}}{2h} \qquad \text{and} \qquad f''(\omega_n) \approx \frac{f_{n+1} + f_{n-1} - f_n}{h^2}.$$

I then approximate the HJB equation as

$$f_n = \min_l \left\{ e^{-\beta(\omega_n;l)\Delta t^h} \left[p_-(\omega_n;l) f_{n-1} + p_+(\omega_n;l) f_{n+1} \right] + (1-l)^{(1-\gamma)(1-\beta_C)} \Delta t^h \right\} \tag{13}$$

where

$$\begin{array}{ll} \beta(\omega_n;l) = u + \beta_C(\gamma-1)c^{-1})e^{\omega_n} l^{1-\beta_I}) & p_-(\omega_n;l) = \frac{1}{2} + h\,\frac{\mu_Y+\delta-\frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega_n} l^{1-\beta_I})}{2\sigma_Y^2} \\ \Delta t^h = \frac{h^2}{\sigma_Y^2} & p_+(\omega_n;l) = \frac{1}{2} - h\,\frac{\mu_Y+\delta-\frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega_n} l^{1-\beta_I})}{2\sigma_Y^2} \end{array}$$

and I have used the approximation $\frac{1}{1+\beta(\omega_n;l)\Delta t^h} \approx e^{-\beta(\omega_n;l)\Delta t^h}$. This corresponds to an Markov Chain approximation to ω, where

$$p(\omega = \omega_n + h | \omega = \omega_n) = p_+(\omega_n;l) \qquad \text{and} \qquad p(\omega = \omega_n - h | \omega = \omega_n) = p_-(\omega_n;l)$$

are the transition probabilities and the time interval is Δt^h. The markov chain is locally consistent because

$$\begin{aligned} E(\Delta\omega_n | \omega_n) &= (\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega_n} l^{1-\beta_I}))\Delta t^h \\ E((\Delta\omega_n - E\Delta\omega_n)^2 | \omega_n) &= \sigma_Y^2 \Delta t^h + o(\Delta t^2) \end{aligned}$$

Note that care must be taken when choosing h to ensure that the probabilities are non-negative for all admissible controls $l \in [0,1]$ at all points in the grid. Alternative differencing schemes that produce positive probabilities can also be used.

Using an initial guess for f, say f^i, one can numerically compute the minimum in (13). Then, given $l_n^{*\,i}$, one can start from $n = 0$ and recursively compute the update on f using the Gauss-Seidel algorithm:

$$f_n^{i+1} = e^{-\beta(\omega_n; l_n^{*\,i})\Delta t^h} \left[p_-(\omega_n; l_n^{*\,i}) f_{n-1}^{i+1} + p_+(\omega_n; l_n^{*\,i}) f_{n+1}^{i} \right] + (1 - l_n^{*\,i})^{(1-\gamma)(1-\beta_C)} \Delta t^h \tag{14}$$

I impose a reflecting barrier on ω at the boundaries of the grid. This reduces to $f_0 = f_1$ and $f_N = f_{N-1}$, since there is no discounting at the boundary and

$$p(\omega = \omega_0 + h|\omega = \omega_0) = 1$$

$$p(\omega = \omega_N - h|\omega = \omega_N) = 1$$

$$\Delta t^h(\omega_N) = \Delta t^h(\omega_0) = 0$$

Finally, because the minimum in (13) is costly to compute, I iterate a couple of times on (14) before updating the policy function.

In the case where $\gamma \neq \theta^{-1}$, the per-period "utility", is also a non-linear function of f:

$$0 = \min_l \left\{ (1-l)^{(1-\gamma)(1-\beta_C)} f(\omega)^c - (u + \beta_C(\gamma-1)c^{-1}(e^\omega l^{1-\beta_I}))f(\omega) + f'(\omega)(\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^\omega l^{1-\beta_I})) + \frac{1}{2}\sigma_Y^2 f''(\omega) \right\}$$

where c is a constant. In this case, my iterations take the form:

$$f_n^{i+1} = e^{-\beta(\omega_n; l_n^{*i})\Delta t^h} \left[p_-(\omega_n; l_n^{*i}) f_{n-1}^{i+1} + p_+(\omega_n; l_n^{*i}) f_{n+1}^i \right] + (1 - l_n^{*i})^{(1-\gamma)(1-\beta_C)} (f_n^i)^c \Delta t^h \tag{15}$$

5 Mapping investment and consumption firms

The procedure is similar to Gomes, Kogan and Yogo (2009). I use the 1997 BEA Standard Make and Use Tables at the detailed level. I use the standard make (table 1) and use (table 2) tables. The uses tables enumerates the contribution of each IO commodity code to Personal Consumption Expenditures (IO code F01000) and Gross Private Fixed Investment (IO code F02000). I use the make tables along with the NAICS-IO map to construct a mapping between 6-digit NAICS Codes to IO commodity codes. I then use the uses table to create a map from IO codes to Investment or Consumption. Because some industries contribute to both PCE and GPFI, I assign industries to the sector they contribute the most value in terms of producer's prices excluding transportation costs.

I use COMPUSTAT to create a PERMNO-NAICS link and form value weighted portfolios using simple returns on all common stocks traded on NYSE, AMEX and Nasdaq. Examples of Investment industries are

IO Code	Description
213111	Drilling oil and gas wells
333111	Farm machinery and equipment manufacturing
333295	Semiconductor machinery manufacturing
334111	Electronic computer manufacturing
334220	Broadcast and wireless communications equipment
336120	Heavy duty truck manufacturing

Examples of Consumption industries are

IO Code	Description
1111B0	Grain farming
221100	Power generation and supply
311410	Frozen food manufacturing
312110	Soft drink and ice manufacturing
325611	Soap and other detergent manufacturing
334300	Audio and video equipment manufacturing

The full list of IO codes and their assignments into industries is available from the author upon request.

6 Additional empirical results and robustness checks

In this section I present a number of additional empirical evidence and robustness tests. In particular, I do the following:

1. I sort firms based on beta-IMC and Book-to-Market Equity with the Consumption and Investment sector separately. I provide these results in Tables 11-16.

2. I repeat the cross-sectional tests including the firms in the investment sector. I examine 10 portfolios of Book-to-market and beta-IMC firms. I first sort firms on BM or beta-IMC within industries. Then I average returns across deciles. So, the p-th portfolio is formed as the average of the p-th portfolios in the C- and I-sector.
$$R_{p,t} = \frac{1}{2}\left(R^C_{p,t} + R^I_{p,t}\right)$$
I show results in Table (17)

3. I consider two alternative measures of I-shocks constructed from the relative price of new equipment. The first is simply minus the growth rate of investment goods prices:
$$z^{\xi,1} = -\Delta \ln \xi$$
the second is the residual $z^{\xi,2}$ from
$$\Delta \ln \xi_t = \rho \ln \xi_{t-1} - z^{\xi,2}_t.$$
I show results in Tables (18)-(19).

4. As and additional test, I can use data on the investment-output ratio and aggregate consumption, to construct estimates of the stochastic discount factor implied by the model (π). It is straightforward to show that I can invert the level of consumption and the investment-output ratio implied by the model to obtain estimates of ω_t and $x_t \equiv X_t K_C^{\beta_C}$. I feed the actual data to the model and compute $\hat{\pi}$
$$\hat{\pi}_t = \frac{h_C(\hat{x}(C_t, i_t), \hat{\omega}(C_t, i_t)}{\frac{1}{T}\sum_{t=1}^T h_C(\hat{x}(C_t, i_t), \hat{\omega}(C_t, i_t)}.$$
The SDF implied by the model ($\hat{\pi}$) has a volatility of 12% per year. I then test that
$$m = a + b_\pi \Delta \ln \hat{\pi}.$$
If the model is correct, then in the above equation $b_\pi = 1$. I show results in Tables (18)-(19).

Table 11: 10 portfolios sorted on IMC beta (Consumption Sector)

IMC Beta	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
					Panel A						
Excess Return (%)	6.54	6.49	7.04	5.48	6.05	5.59	3.65	2.67	4.64	4.40	-2.14
	(2.63)	(3.01)	(3.24)	(2.45)	(2.53)	(2.25)	(1.38)	(0.85)	(1.29)	(0.96)	(-0.53)
σ (%)	16.70	14.48	14.57	14.96	16.03	16.71	17.75	21.08	24.17	30.92	26.85
Sharpe Ratio (%)	39.16	44.81	48.33	36.59	37.73	33.47	20.55	12.65	19.21	14.24	-7.96
β_{MKT}	0.81	0.78	0.83	0.86	0.96	0.99	1.06	1.23	1.39	1.64	0.83
	(17.94)	(26.45)	(29.27)	(30.69)	(48.75)	(46.79)	(51.57)	(43.62)	(34.59)	(25.15)	(8.71)
α(%)	2.94	3.03	3.37	1.68	1.81	1.21	-1.02	-2.78	-1.49	-2.86	-5.80
	(1.74)	(2.56)	(3.11)	(1.55)	(2.04)	(1.26)	(-0.99)	(-2.14)	(-0.87)	(-1.13)	(-1.89)
R^2(%)	56.97	69.96	77.94	78.97	85.66	84.59	85.13	82.01	79.11	67.77	22.90
					Panel B						
β_{MKT}	0.94	0.87	0.91	0.94	1.00	1.01	1.05	1.15	1.25	1.33	0.39
	(22.76)	(31.39)	(29.74)	(33.93)	(50.38)	(35.67)	(42.84)	(40.40)	(30.66)	(23.20)	(4.85)
β_{SMB}	-0.05	-0.26	-0.16	-0.19	-0.07	0.02	0.03	0.17	0.24	0.63	0.68
	(-0.83)	(-7.25)	(-4.95)	(-4.66)	(-2.15)	(0.40)	(0.85)	(3.75)	(4.19)	(7.98)	(6.19)
β_{HML}	0.44	0.13	0.19	0.15	0.09	0.08	-0.02	-0.19	-0.37	-0.72	-1.16
	(5.95)	(2.56)	(3.99)	(2.65)	(2.23)	(1.65)	(-0.54)	(-3.26)	(-5.53)	(-6.96)	(-7.70)
α(%)	0.31	2.75	2.54	1.14	1.42	0.64	-0.95	-1.96	0.30	0.30	-0.00
	(0.20)	(2.33)	(2.49)	(1.16)	(1.57)	(0.64)	(-0.86)	(-1.42)	(0.19)	(0.14)	(-0.00)
R^2(%)	63.41	74.83	81.20	81.99	86.21	84.80	85.19	83.64	82.65	78.57	49.82

Table 11 reports summary statistics of the simple monthly returns of 10 portfolios created by sorting stocks on their univariate beta with respect to the IMC portfolio (β^{IMC}). I focus only on firms in the consumption-goods industry. I sort firms into portfolios based on their pre-ranking beta with the IMC portfolio. I estimate pre-ranking betas using weekly data and a window of one year. I rebalance portfolios at the end of every December. Panel A reports mean excess returns over the 30-day T-bill rate (μ), the standard deviation of returns (σ) and their beta and alpha with respect to the CAPM. Panel B reports beta and alpha for the Fama and French (1993) three factor model. Panel C reports beta and alpha for the model including the market portfolio and the IMC portfolio. t-statistics based on HAC standard errors with 1 lag are shown in parenthesis. I use monthly data in the 1963:2008 period.

Table 12: 10 PORTFOLIOS SORTED ON IMC BETA (INVESTMENT SECTOR)

IMC Beta	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
						Panel A					
Excess Return (%)	8.57	9.29	6.46	7.08	6.62	7.19	6.55	3.48	3.36	3.17	-5.40
	(2.54)	(3.27)	(2.32)	(2.20)	(2.01)	(2.07)	(1.82)	(0.85)	(0.76)	(0.62)	(-1.21)
σ (%)	22.61	19.08	18.68	21.55	22.09	23.31	24.13	27.58	29.63	34.51	29.81
Sharpe Ratio (%)	37.90	48.68	34.59	32.87	29.97	30.86	27.17	12.63	11.33	9.19	-18.11
β_{MKT}	0.97	1.01	1.03	1.16	1.22	1.27	1.30	1.44	1.50	1.74	0.77
	(15.48)	(26.66)	(29.47)	(29.19)	(38.21)	(31.58)	(31.15)	(23.80)	(23.40)	(20.93)	(6.68)
α(%)	4.29	4.84	1.89	1.94	1.25	1.60	0.81	-2.86	-3.25	-4.49	-8.79
	(1.64)	(2.89)	(1.28)	(1.14)	(0.71)	(0.89)	(0.40)	(-1.14)	(-1.17)	(-1.34)	(-2.06)
R^2(%)	43.91	66.59	73.49	69.96	72.65	70.66	69.71	64.93	61.07	60.59	15.87
						Panel B					
β_{MKT}	0.97	1.01	1.04	1.08	1.12	1.06	1.16	1.17	1.24	1.42	0.45
	(14.68)	(32.84)	(34.01)	(24.01)	(32.46)	(21.89)	(28.41)	(24.58)	(19.35)	(18.59)	(4.34)
β_{SMB}	0.54	0.37	0.29	0.43	0.37	0.49	0.25	0.35	0.29	0.49	-0.05
	(4.43)	(7.06)	(5.89)	(4.54)	(5.61)	(5.38)	(3.65)	(4.85)	(2.50)	(5.08)	(-0.34)
β_{HML}	0.47	0.34	0.27	0.03	-0.05	-0.39	-0.34	-0.75	-0.76	-0.84	-1.32
	(4.41)	(5.45)	(4.93)	(0.40)	(-0.74)	(-4.74)	(-4.21)	(-7.38)	(-6.36)	(-6.83)	(-7.85)
α(%)	0.23	1.98	-0.39	0.84	0.78	3.02	2.39	1.07	0.86	-0.27	-0.50
	(0.09)	(1.27)	(-0.28)	(0.47)	(0.45)	(1.79)	(1.24)	(0.50)	(0.34)	(-0.09)	(-0.13)
R^2(%)	52.34	72.41	77.22	74.27	76.03	79.26	73.03	74.27	68.59	69.16	32.50

Table 12 reports summary statistics of the simple monthly returns of 10 portfolios created by sorting stocks on their univariate beta with respect to the IMC portfolio (β^{IMC}). I focus only on firms in the investment-goods industry. I sort firms into portfolios based on their pre-ranking beta with the IMC portfolio. I estimate pre-ranking betas using weekly data and a window of one year. I rebalance portfolios at the end of every December. Panel A reports mean excess returns over the 30-day T-bill rate (μ), the standard deviation of returns (σ) and their beta and alpha with respect to the CAPM. Panel B reports beta and alpha for the Fama and French (1993) three factor model. Panel C reports beta and alpha for the model including the market portfolio and the IMC portfolio. t-statistics based on HAC standard errors with 1 lag are shown in parenthesis. I use monthly data in the 1963:2008 period.

Table 13: 10 PORTFOLIOS SORTED ON BOOK-TO-MARKET (CONSUMPTION SECTOR)

BE/ME	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
						Panel A					
Excess Return (%)	3.31	5.43	4.20	5.19	5.30	5.90	5.50	7.95	7.74	8.90	5.59
	(1.31)	(2.29)	(1.76)	(2.35)	(2.33)	(2.63)	(2.37)	(3.20)	(2.99)	(3.06)	(2.52)
σ (%)	16.98	15.90	16.00	14.79	15.24	15.03	15.57	16.63	17.38	19.54	14.85
Sharpe Ratio (%)	19.52	34.13	26.24	35.09	34.80	39.24	35.32	47.78	44.56	45.57	37.63
β_{MKT}	1.01	0.97	0.97	0.87	0.89	0.85	0.87	0.92	0.95	1.05	0.04
	(43.59)	(48.22)	(50.35)	(34.56)	(33.90)	(26.77)	(23.98)	(24.91)	(24.03)	(23.08)	(0.74)
α(%)	-1.15	1.13	-0.08	1.34	1.39	2.14	1.66	3.90	3.53	4.25	5.40
	(-1.11)	(1.43)	(-0.10)	(1.38)	(1.41)	(1.97)	(1.33)	(2.84)	(2.44)	(2.50)	(2.31)
R^2(%)	84.79	89.49	87.72	83.15	80.99	76.85	74.77	72.71	72.29	69.70	0.21
						Panel B					
β_{MKT}	0.96	0.98	1.02	0.95	0.99	0.99	1.02	1.07	1.10	1.16	0.19
	(47.74)	(48.01)	(47.51)	(37.53)	(36.60)	(41.24)	(48.95)	(42.16)	(44.70)	(37.80)	(5.45)
β_{SMB}	-0.21	-0.12	-0.08	-0.12	-0.09	-0.09	0.00	0.10	0.16	0.43	0.64
	(-6.54)	(-4.77)	(-1.93)	(-3.92)	(-2.44)	(-2.63)	(0.10)	(3.19)	(4.72)	(10.83)	(12.80)
β_{HML}	-0.38	-0.08	0.13	0.22	0.33	0.48	0.61	0.71	0.74	0.78	1.16
	(-9.21)	(-2.04)	(2.93)	(5.38)	(7.12)	(11.37)	(17.31)	(20.05)	(18.39)	(16.18)	(18.47)
α(%)	1.63	1.86	-0.73	0.21	-0.47	-0.65	-2.12	-0.72	-1.42	-1.50	-3.13
	(1.90)	(2.31)	(-0.92)	(0.23)	(-0.50)	(-0.84)	(-2.60)	(-0.80)	(-1.40)	(-1.31)	(-2.17)
R^2(%)	89.81	90.20	88.71	86.26	85.85	86.56	87.82	87.96	87.55	85.91	61.94

Table 13 reports summary statistics of the simple monthly returns of 10 portfolios created by sorting stocks on their book-to-market ratio (Compustat Item 60 divided by end of year market capitalization). I focus only on firms in the consumption-goods industry. Panel A reports mean excess returns over the 30-day T-bill rate (μ), the standard deviation of returns (σ) and their beta and alpha with respect to the CAPM. Panel B reports beta and alpha for the Fama and French (1993) three factor model. Panel C reports beta and alpha for the model including the market portfolio and the IMC portfolio. t-statistics based on HAC standard errors with 1 lag are shown in parenthesis. I use monthly data in the 1963:2008 period.

Table 14: 10 PORTFOLIOS SORTED ON BOOK-TO-MARKET (INVESTMENT SECTOR)

BE/ME	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
					Panel A						
Excess Return (%)	4.71	6.74	4.69	6.29	6.63	7.00	11.12	11.03	9.66	10.80	6.09
	(1.17)	(1.86)	(1.36)	(1.90)	(1.95)	(2.10)	(3.36)	(3.24)	(2.71)	(3.01)	(1.73)
σ (%)	27.02	24.27	23.14	22.19	22.78	22.33	22.20	22.85	23.97	24.06	23.65
Sharpe Ratio (%)	17.43	27.76	20.27	28.33	29.11	31.36	50.08	48.27	40.32	44.87	25.75
β_{MKT}	1.39	1.32	1.28	1.20	1.25	1.20	1.17	1.22	1.21	1.13	-0.26
	(26.60)	(36.11)	(31.80)	(31.81)	(33.38)	(26.59)	(27.75)	(23.42)	(24.13)	(18.49)	(-2.83)
α(%)	-1.44	0.89	-0.97	0.99	1.12	1.69	5.95	5.65	4.33	5.80	7.24
	(-0.58)	(0.45)	(-0.53)	(0.52)	(0.61)	(0.94)	(2.95)	(2.92)	(1.86)	(2.33)	(2.01)
R^2(%)	63.64	71.19	73.42	70.06	71.98	69.46	66.41	68.15	60.79	53.00	2.91
					Panel B						
β_{MKT}	1.12	1.12	1.14	1.10	1.18	1.16	1.11	1.17	1.17	1.14	0.02
	(22.93)	(30.29)	(31.29)	(27.51)	(27.93)	(28.93)	(32.11)	(22.91)	(26.51)	(27.36)	(0.41)
β_{SMB}	0.20	0.28	0.29	0.37	0.39	0.44	0.58	0.55	0.69	0.84	0.64
	(2.42)	(4.92)	(4.77)	(6.30)	(5.05)	(7.11)	(9.65)	(6.62)	(7.85)	(9.99)	(7.28)
β_{HML}	-0.93	-0.56	-0.30	-0.07	0.07	0.22	0.29	0.30	0.44	0.77	1.69
	(-11.89)	(-8.79)	(-3.48)	(-0.78)	(0.93)	(3.21)	(4.22)	(3.57)	(5.12)	(8.77)	(18.02)
α(%)	3.90	3.81	0.27	0.63	-0.15	-0.57	2.97	2.64	0.14	-0.72	-4.62
	(1.85)	(2.15)	(0.15)	(0.37)	(-0.08)	(-0.34)	(1.63)	(1.50)	(0.07)	(-0.37)	(-1.75)
R^2(%)	75.16	78.27	77.06	73.43	75.10	73.91	74.29	75.14	71.33	71.58	48.75

Table 14 reports summary statistics of the simple monthly returns of 10 portfolios created by sorting stocks on their book-to-market ratio (Compustat Item 60 divided by end of year market capitalization). I focus only on firms in the investment-goods industry. Panel A reports mean excess returns over the 30-day T-bill rate (μ), the standard deviation of returns (σ) and their beta and alpha with respect to the CAPM. Panel B reports beta and alpha for the Fama and French (1993) three factor model. Panel C reports beta and alpha for the model including the market portfolio and the IMC portfolio. t-statistics based on HAC standard errors with 1 lag are shown in parenthesis. I use monthly data in the 1963:2008 period.

Table 15: Summary Statistics: Excess Return of 10 portfolios sorted on IMC beta and Book-to-Market

	A: 10 portfolios sorted on IMC beta (Consumption firms only)										
IMC Beta	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
Mean (μ)	6.47	6.11	6.61	6.62	6.14	5.77	5.37	5.36	5.27	4.36	-2.11
	(2.34)	(2.25)	(2.68)	(2.87)	(2.28)	(2.01)	(1.88)	(1.63)	(1.30)	(0.88)	(-0.46)
Volatility (σ)	18.52	18.19	16.55	15.44	18.07	19.27	19.16	22.01	27.24	33.20	30.70
Sharpe Ratio (μ/σ)	34.92	33.59	39.98	42.84	33.99	29.94	28.01	24.37	19.35	13.13	-6.87
$cov(R_i, R_{mkt})/\sigma^2_{mkt}$	0.80	0.89	0.80	0.75	0.96	1.03	1.01	1.12	1.34	1.48	0.68
	(6.60)	(11.73)	(11.76)	(9.07)	(15.22)	(20.40)	(34.78)	(20.48)	(12.21)	(7.48)	(2.23)
$cov(R_i, \Delta \ln C)/\sigma^2_c$	3.51	2.25	2.46	1.57	3.03	3.16	4.15	3.33	4.86	6.99	3.48
	(2.20)	(1.11)	(1.75)	(0.94)	(1.90)	(1.74)	(2.19)	(1.77)	(1.74)	(2.00)	(0.99)
$cov(R_i, R_{imc})/\sigma^2_{imc}$	-0.07	-0.03	-0.07	-0.11	0.23	0.25	0.60	0.61	1.25	1.77	1.84
	(-0.36)	(-0.21)	(-0.69)	(-0.95)	(1.89)	(1.60)	(5.68)	(2.80)	(9.92)	(10.27)	(10.67)
$cov(R_i, z^\xi)/\sigma^2_z$	-3.41	-2.80	-2.60	-2.37	-2.77	-3.18	-1.67	-2.47	-1.38	-0.51	2.90
	(-2.47)	(-2.38)	(-2.60)	(-2.67)	(-2.16)	(-2.40)	(-1.06)	(-1.53)	(-0.70)	(-0.20)	(2.14)
	B: 10 portfolios sorted on book-to-market (Consumption firms only)										
BE/ME	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
Mean (μ)	3.88	5.93	4.66	5.63	5.86	6.79	6.60	8.89	9.18	10.04	6.15
	(1.29)	(2.25)	(1.73)	(2.26)	(2.22)	(2.45)	(2.28)	(3.04)	(2.77)	(2.98)	(2.16)
Volatility (σ)	20.16	17.68	18.09	16.73	17.72	18.57	19.45	19.60	22.20	22.62	19.12
Sharpe Ratio (μ/σ)	19.27	33.54	25.75	33.68	33.08	36.57	33.92	45.37	41.33	44.37	32.17
$cov(R_i, R_{mkt})/\sigma^2_{mkt}$	1.01	0.91	0.94	0.85	0.88	0.89	0.91	0.92	1.00	1.03	0.02
	(18.68)	(26.87)	(13.83)	(11.64)	(10.53)	(8.31)	(7.21)	(8.27)	(9.46)	(9.52)	(0.16)
$cov(R_i, \Delta \ln C)/\sigma^2_c$	2.40	1.99	2.39	2.83	2.73	3.18	4.05	2.67	3.59	3.42	1.02
	(1.07)	(1.19)	(1.33)	(1.81)	(1.66)	(1.73)	(2.27)	(1.65)	(1.74)	(1.61)	(0.69)
$cov(R_i, R_{imc})/\sigma^2_{imc}$	0.38	0.23	0.23	0.23	0.05	0.06	-0.00	-0.05	0.02	0.19	-0.19
	(2.61)	(1.84)	(1.44)	(1.48)	(0.33)	(0.30)	(-0.02)	(-0.33)	(0.12)	(1.17)	(-1.24)
$cov(R_i, z^\xi)/\sigma^2_z$	-2.36	-2.14	-2.41	-2.60	-2.47	-3.38	-3.05	-3.30	-3.62	-3.35	-0.99
	(-1.68)	(-1.66)	(-1.55)	(-2.11)	(-2.24)	(-2.06)	(-2.41)	(-2.38)	(-3.11)	(-2.38)	(-1.69)

Table **??** reports summary statistics for simple excess returns over the 30-day T-bill rate for two sets of portfolios created by using only firms in the consumption industry. I report mean returns (μ) and volatilities (σ). I also report univariate betas with respect to my two measures for the C-shock: returns on the market portfolio (R_{mkt}) and consumption growth ($\Delta \ln C$), univariate betas with respect to my two measures for the I-shock: minus the innovation in the relative price of new equipment (z^ξ), and returns on the portfolio of investment to consumption producers (R_{imc}). The relative price of investment (ξ) comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I compute innovations z^ξ_t from $\ln \xi_t = a_0\, t + a_1\, t \cdot 1_{t>1982} + \rho \ln \xi_{t-1} - z^\xi_t$. Panel A shows results for 10 portfolios created by sorting stocks on their univariate beta with respect to the IMC portfolio (β^{IMC}). I sort firms into portfolios based on their pre-ranking beta with the IMC portfolio. I estimate pre-ranking betas using weekly data and a window of one year. I rebalance portfolios at the end of every December. Panel B shows results for 10 portfolios created by sorting stocks on their book-to-market ratio. I focus only on firms in the consumption industry. I sort firms into 10 portfolios using NYSE breakpoints. I rebalance portfolios every June. Sample includes annual data from 1963:2008.

Table 16: SUMMARY STATISTICS: EXCESS RETURN OF 10 PORTFOLIOS SORTED ON IMC BETA AND BOOK-TO-MARKET

	A: 10 portfolios sorted on IMC beta (Investment firms only)										
IMC Beta	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
Mean (μ)	9.06	10.12	6.86	8.13	7.19	7.54	7.64	4.58	3.76	3.98	-5.08
	(2.50)	(3.19)	(2.28)	(2.11)	(1.98)	(2.03)	(1.70)	(0.95)	(0.78)	(0.71)	(-0.92)
Volatility (σ)	24.36	21.27	20.16	25.81	24.31	24.92	30.05	32.22	32.40	37.69	37.00
Sharpe Ratio (μ/σ)	37.21	47.55	34.02	31.51	29.59	30.26	25.41	14.21	11.60	10.57	-13.73
$cov(R_i, R_{mkt})/\sigma^2_{mkt}$	0.75	0.96	0.87	1.11	1.12	1.15	1.37	1.36	1.26	1.52	0.78
	(4.20)	(8.20)	(7.94)	(7.57)	(9.93)	(12.57)	(10.97)	(6.02)	(6.53)	(7.05)	(2.39)
$cov(R_i, \Delta \ln C)/\sigma^2_c$	-0.87	2.77	4.48	3.57	2.74	4.79	5.68	6.40	6.87	8.56	9.43
	(-0.36)	(1.42)	(2.58)	(1.46)	(1.14)	(2.17)	(2.19)	(1.97)	(2.08)	(2.45)	(2.68)
$cov(R_i, R_{imc})/\sigma^2_{imc}$	0.01	0.11	0.32	0.41	0.46	1.16	0.94	1.75	1.88	1.83	1.83
	(0.02)	(0.43)	(0.97)	(0.98)	(1.62)	(5.54)	(3.34)	(10.35)	(9.35)	(5.56)	(8.37)
$cov(R_i, z^{\xi})/\sigma^2_z$	-2.29	-2.69	-3.02	-3.68	-3.85	-3.02	-1.35	-0.05	-0.66	0.51	2.80
	(-1.19)	(-2.18)	(-2.31)	(-1.93)	(-2.95)	(-1.96)	(-0.54)	(-0.02)	(-0.30)	(0.17)	(1.07)
	B: 10 portfolios sorted on book-to-market (Investment firms only)										
BE/ME	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
Mean (μ)	6.16	7.67	5.10	6.52	7.53	7.11	12.23	12.13	11.20	12.71	5.04
	(1.20)	(1.72)	(1.34)	(1.89)	(1.85)	(2.17)	(3.24)	(3.13)	(2.46)	(2.68)	(0.93)
Volatility (σ)	34.32	30.01	25.53	23.17	27.24	21.96	25.32	26.02	30.57	31.83	36.35
Sharpe Ratio (μ/σ)	17.96	25.57	19.99	28.13	27.63	32.38	48.28	46.63	36.63	39.95	13.85
$cov(R_i, R_{mkt})/\sigma^2_{mkt}$	1.40	1.17	1.24	1.04	1.24	1.01	0.94	1.15	1.19	1.25	-0.21
	(7.32)	(7.01)	(13.22)	(13.83)	(9.61)	(8.80)	(5.52)	(8.78)	(5.80)	(6.52)	(-0.60)
$cov(R_i, \Delta \ln C)/\sigma^2_c$	5.51	5.23	4.66	5.06	5.30	5.56	4.13	3.27	2.58	3.58	-2.93
	(1.38)	(1.89)	(1.98)	(2.84)	(2.68)	(3.02)	(1.65)	(1.59)	(1.01)	(1.60)	(-0.75)
$cov(R_i, R_{imc})/\sigma^2_{imc}$	1.60	1.45	1.05	0.85	0.90	0.58	0.42	0.51	0.03	0.34	-1.58
	(8.54)	(5.78)	(4.84)	(4.46)	(2.41)	(1.91)	(1.15)	(2.06)	(0.06)	(0.88)	(-4.78)
$cov(R_i, z^{\xi})/\sigma^2_z$	0.64	-0.42	-2.36	-1.11	-4.63	-3.01	-3.95	-3.05	-3.33	-5.14	-3.97
	(0.23)	(-0.20)	(-1.05)	(-0.65)	(-2.09)	(-1.76)	(-3.05)	(-2.07)	(-2.47)	(-2.40)	(-2.00)

Table 12 reports summary statistics for simple excess returns over the 30-day T-bill rate for two sets of portfolios created by using only firms in the consumption industry. I report mean returns (μ) and volatilities (σ). I also report univariate betas with respect to my two measures for the C-shock: returns on the market portfolio (R_{mkt}) and consumption growth ($\Delta \ln C$), univariate betas with respect to my two measures for the I-shock: minus the innovation in the relative price of new equipment (z^{ξ}), and returns on the portfolio of investment to consumption producers (R_{imc}). The relative price of investment (ξ) comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I compute innovations z^{ξ}_t from $\ln \xi_t = a_0\, t + a_1\, t \cdot 1_{t>1982} + \rho \ln \xi_{t-1} - z^{\xi}_t$. Panel A shows results for 10 portfolios created by sorting stocks on their univariate beta with respect to the IMC portfolio (β^{IMC}). I sort firms into portfolios based on their pre-ranking beta with the IMC portfolio. I estimate pre-ranking betas using weekly data and a window of one year. I rebalance portfolios at the end of every December. Panel B shows results for 10 portfolios created by sorting stocks on their book-to-market ratio. I focus only on firms in the consumption industry. I sort firms into 10 portfolios using NYSE breakpoints. I rebalance portfolios every June. Sample includes annual data from 1963:2008.

Table 17: CROSS-SECTIONAL TESTS (ALL INDUSTRIES)

Panel A: 10 β^{IMC} sorted portfolios (within industries)							
Factor	CAPM	CCAPM	IMC, MKT	IMC, C	P_I, MKT	P_I, C	$\hat{\pi}$
$\Delta\tilde{x}$ (MKT)	1.66		2.95		0.57		
	[2.38]		[4.84]		[0.59]		
$\Delta\tilde{x}$ (C)		47.92		137.76		13.20	
		[2.08]		[4.95]		[0.56]	
$\Delta\tilde{z}$ (IMC)			-3.49	-6.85			
			[-3.11]	[-5.28]			
$\Delta\tilde{z}$ (z^{ξ})					-41.88	-48.17	
					[-2.92]	[-5.25]	
$\hat{\pi}$							45.58
							[3.94]
$SSQE(\%)$	0.74	1.54	0.08	0.45	0.12	0.14	1.22
J-test	19.0 [0.03]	47.3 [0.00]	9.7 [0.29]	37.9 [0.00]	16.9 [0.03]	24.3 [0.00]	22.9 [0.00]
Panel B: 10 Book-to-Market sorted portfolios (within industries)							
Factor	CAPM	CCAPM	IMC, MKT	IMC, C	P_I, MKT	P_I, C	$\hat{\pi}$
$\Delta\tilde{x}$ (MKT)	2.27		3.25		0.95		
	[3.22]		[3.63]		[1.03]		
$\Delta\tilde{x}$ (C)		78.11		108.33		23.80	
		[3.21]		[3.59]		[0.81]	
$\Delta\tilde{z}$ (IMC)			-4.34	-3.81			
			[-2.17]	[-1.92]			
$\Delta\tilde{z}$ (z^{ξ})					-37.11	-43.82	
					[-1.95]	[-2.37]	
$\hat{\pi}$							46.49
							[3.47]
$SSQE(\%)$	0.56	0.79	0.16	0.48	0.22	0.29	0.73
J-test	39.5 [0.00]	62.2 [0.00]	32.7 [0.00]	61.4 [0.00]	38.3 [0.00]	60.9 [0.00]	58.3 [0.00]

Table 17 shows results of estimating the stochastic discount factor of the model via generalized method of moments. I report first stage estimates of b_X, b_Z and b_π using the identity weighting matrix. I also report the sum of squared pricing errors (SSQE) and the J test of overidentifying restrictions along with p-values in brackets. Standard errors are computed using the Newey-West estimator allowing for 1 lag. I use two proxies for X: returns on the market portfolio (MKT) and the growth rate of per capita non-durables plus services consumption (C). I use two proxies for Z: returns on the IMC portfolio and minus the innovations in the relative price of investment goods (ξ). The relative price of investment (ξ) comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I compute innovations z^{ξ}_t from $\ln \xi_t = a_0\, t + a_1\, t \cdot 1_{t>1982} + \rho \ln \xi_{t-1} - z^{\xi}_t$. I construct $\hat{\pi}$ using the full solution of the model and consumption growth and the investment-output ratio (non-residential fixed investment divided by GDP excluding government spending). When sorting firms on characteristics, I sort them first within their industry (consumption or investment). I then form the decile portfolios by averaging them across industries. The top panel (A), shows estimates of the parameters using a cross-section of 10 portfolios sorted on their univariate beta with respect to the IMC portfolio (β^{IMC}). The bottom panel (B) shows estimates of the parameters using a cross-section of 10 portfolios sorted on their book-to-market ratio. Sample includes annual data from 1963:2008.

Table 18: Alternative measures of I-shock: Cross-Sectional Tests (I)

Panel A: 10 β^{IMC} sorted portfolios (Consumption Sector only)							
Factor	CAPM	CCAPM	$P_I(I), MKT$	$P_I(I), C$	$P_I(II), MKT$	$P_I(II), C$	$\hat{\pi}$
$\Delta\tilde{x}$ (MKT)	1.64		0.92		0.89		
	[2.20]		[0.96]		[0.92]		
$\Delta\tilde{x}$ (C)		51.54		24.05		23.57	
		[2.05]		[0.90]		[0.88]	
$\Delta\tilde{z}$ $(-\Delta\ln\xi)$			-30.97	-39.57			
			[-1.61]	[-2.58]			
$\Delta\tilde{z}$ (z^{ξ})					-31.98	-40.04	
					[-1.63]	[-2.57]	
$\hat{\pi}$							34.95
							[2.79]
$SSQE(\%)$	0.37	0.78	0.10	0.14	0.07	0.10	0.59
J-test	12.3 [0.20]	24.4 [0.00]	12.1 [0.15]	18.1 [0.02]	11.5 [0.17]	15.8 [0.04]	22.3 [0.00]

Panel B: 10 Book-to-Market sorted portfolios (Consumption Sector only)							
Factor	CAPM	CCAPM	$P_I(I), MKT$	$P_I(I), C$	$P_I(II), MKT$	$P_I(II), C$	$\hat{\pi}$
$\Delta\tilde{x}$ (MKT)	2.23		0.47		0.09		
	[3.06]		[0.46]		[0.07]		
C		87.46		8.49		-9.30	
		[3.09]		[0.23]		[-0.35]	
$\Delta\tilde{z}$ $(-\Delta\ln\xi)$			-47.68	-53.94			
			[-2.05]	[-2.95]			
$\Delta\tilde{z}$ (z^{ξ})					-56.65	-64.90	
					[-1.99]	[-4.60]	
$\hat{\pi}$							37.13
							[3.12]
$SSQE(\%)$	0.33	0.27	0.10	0.12	0.12	0.12	0.33
J-test	53.3 [0.00]	63.4 [0.00]	38.6 [0.00]	27.7 [0.00]	37.3 [0.00]	25.6 [0.00]	63.5 [0.00]

Table 18 shows results of estimating the stochastic discount factor of the model via generalized method of moments. I report first stage estimates of b_X, b_Z and b_π using the identity weighting matrix. I also report the sum of squared pricing errors (SSQE) and the J test of overidentifying restrictions along with p-values in brackets. Standard errors are computed using the Newey-West estimator allowing for 1 lag. I use two proxies for X: returns on the market portfolio (MKT) and the growth rate of per capita non-durables plus services consumption (C). I use two proxies for Z: minus the change in the relative price of new equipment $(-\Delta\ln\xi)$ and minus the innovations in the relative price of new equipment (z^{ξ}_t). The relative price of investment (ξ) comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I compute innovations from $\Delta\ln\xi_t = \rho\ln\xi_{t-1} - z^{\xi}_t$. The top panel (A), shows estimates of the parameters using a cross-section of 10 portfolios sorted on their univariate beta with respect to the IMC portfolio (β^{IMC}). I focus only on firms in the consumption industry. I sort firms into portfolios based on their pre-ranking beta with the IMC portfolio. I estimate pre-ranking betas using weekly data and a window of one year. I rebalance portfolios at the end of every December. The bottom panel (B) shows estimates of the parameters using a cross-section of 10 portfolios sorted on their book-to-market ratio. I focus only on firms in the consumption industry. I sort firms into 10 portfolios using NYSE breakpoints. I rebalance portfolios every June. Sample includes annual data from 1963:2008.

Figure 6: Historical average returns versus covariances with I-shock

A: 10 β^{IMC} sorted portfolios (Consumption Sector only)

B: 10 Book-to-Market sorted portfolios (Consumption Sector only)

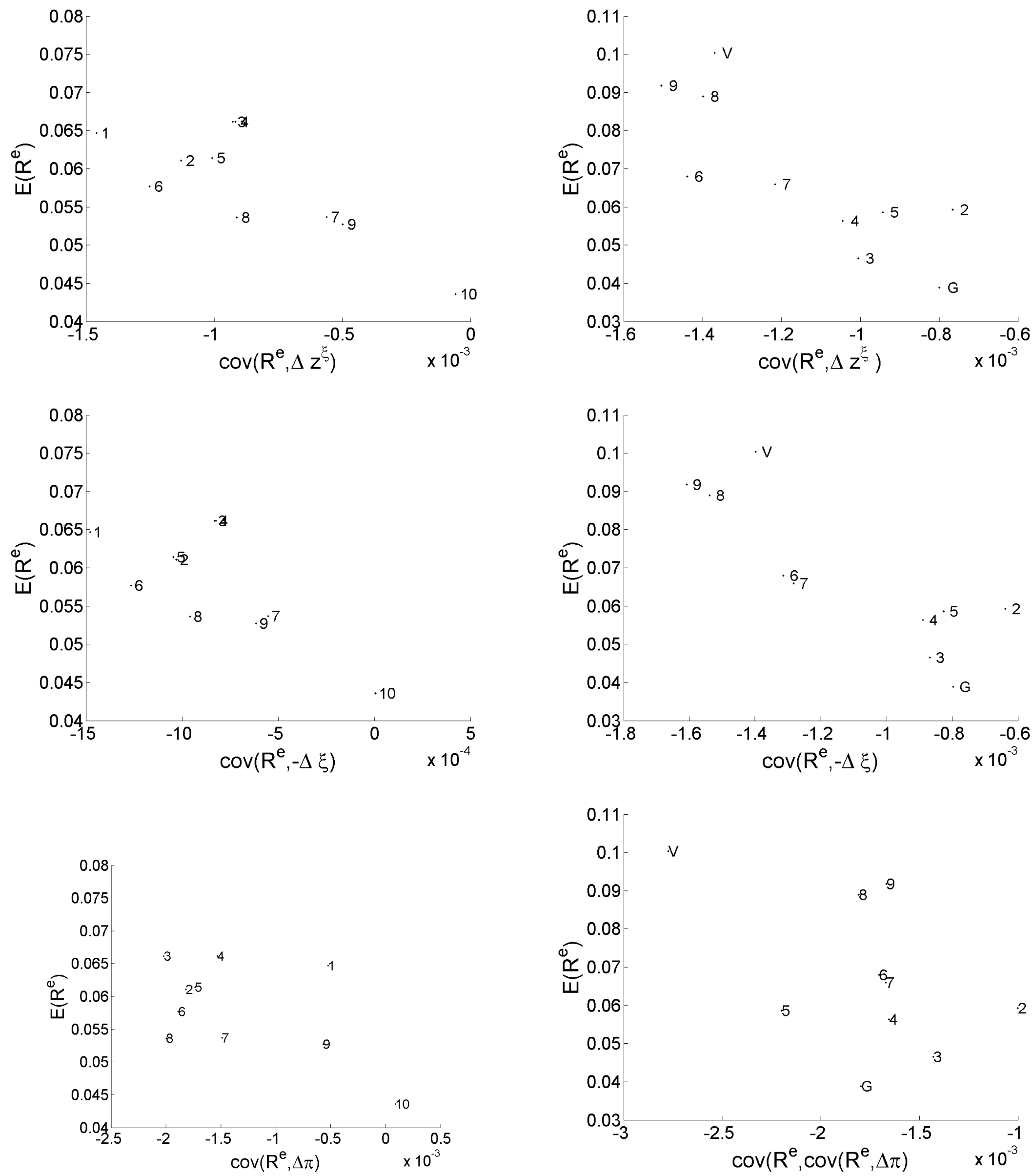


Figure 6 plots average returns versus covariances with the investment-specific shock (Z) and $\hat{\pi}$ for two sets of portfolios. I use two proxies for Z: first differences and innovations in the relative price of investment goods (ξ). The relative price of investment (ξ) comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I compute innovations ξ_t^ϵ from $\ln \xi_t = a_0\, t + \rho \ln \xi_{t-1} + \xi_t^\epsilon$. I allow the time trend to vary before and after 1982 following the findings of Fisher (2006). I construct $\hat{\pi}$ using the full solution of the model and consumption growth and the investment-output ratio (non-residential fixed investment divided by GDP excluding government spending). The left panel (A), uses 10 portfolios sorted on their univariate beta with respect to the IMC portfolio (β^{IMC}). I include only consumption firms. I sort firms into portfolios based on their pre-ranking beta with the IMC portfolio. I estimate pre-ranking betas using weekly data and a window of one year. I rebalance portfolios at the end of every December. The right panel (B) uses 10 portfolios sorted on their book-to-market ratio. I include only consumption firms. I sort firms into 10 portfolios using NYSE breakpoints. I rebalance portfolios every June. Sample includes annual data from 1963:2008.

Table 19: ALTERNATIVE MEASURES OF I-SHOCK: CROSS-SECTIONAL TESTS (II)

Panel A: 25 Market Equity/Book-to-Market sorted portfolios							
Factor	CAPM	CCAPM	$P_I(I), MKT$	$P_I(I), C$	$P_I(II), MKT$	$P_I(II), C$	$\hat{\pi}$
$\Delta\tilde{x}$ (MKT)	2.42		0.38		-0.22		
	[3.34]		[0.46]		[-0.30]		
$\Delta\tilde{x}$ (C)		107.56		52.69		43.55	
		[3.59]		[2.02]		[1.69]	
$\Delta\tilde{z}$ $(-\Delta \ln \xi)$			-46.33	-30.12			
			[-2.58]	[-1.66]			
$\Delta\tilde{z}$ (z^{ξ})					-64.26	-37.22	
					[-3.99]	[-1.86]	
$\hat{\pi}$							38.68
							[3.35]
$SSQE(\%)$	3.69	3.05	2.81	2.12	2.45	2.06	0.99
J-test	113.4 [0.00]	130.8 [0.00]	112.2 [0.00]	122.6 [0.00]	111.4 [0.00]	120.1 [0.00]	130.2 [0.00]
Panel B: 30 Industry Portfolios							
Factor	CAPM	CCAPM	$P_I(I), MKT$	$P_I(I), C$	$P_I(II), MKT$	$P_I(II), C$	$\hat{\pi}$
$\Delta\tilde{x}$ (MKT)	2.02		1.86		1.73		
	[2.31]		[2.25]		[2.04]		
$\Delta\tilde{x}$ (C)		65.26		37.17		34.54	
		[2.81]		[2.07]		[1.93]	
$\Delta\tilde{z}$ $(-\Delta \ln \xi)$			-3.54	-24.01			
			[-0.33]	[-2.49]			
$\Delta\tilde{z}$ (z^{ξ})					-6.71	-26.88	
					[-0.58]	[-2.62]	
$\hat{\pi}$							20.47
							[2.87]
$SSQE(\%)$	2.38	5.83	2.36	2.86	2.31	2.62	6.47
J-test	117.1 [0.00]	150.6 [0.00]	128.8 [0.00]	140.4 [0.00]	129.1 [0.00]	138.7 [0.00]	148.7 [0.00]

Table 19 shows results of estimating the stochastic discount factor of the model via generalized method of moments. I report first stage estimates of b_X and b_Z using the identity weighting matrix. I also report the sum of squared pricing errors (SSQE) and the J test of overidentifying restrictions along with p-values in brackets.Standard errors are computed using the Newey-West estimator allowing for 1 lag. I use two proxies for X: returns on the market portfolio (MKT) and the growth rate of per capita non-durables plus services consumption (C). I use two proxies for Z: minus the change in the relative price of new equipment $(-\Delta \ln \xi)$ and minus the innovations in the relative price of new equipment (z^{ξ}_t). The relative price of investment (ξ) comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I compute innovations from $\Delta \ln \xi_t = \rho \ln \xi_{t-1} - z^{\xi}_t$. The top panel (A), shows estimates of the parameters using the cross-section of 25 portfolios sorted on market-equity and book-to-market from Fama and French (1993). The bottom panel (B) shows results using the cross-section of 30 industry portfolios of Fama and French (1997).

Figure 7: Historical average returns versus covariances with I-shock

A: 25 Market-Equity/Book-to-Market sorted portfolios

B: 30 Industry portfolios

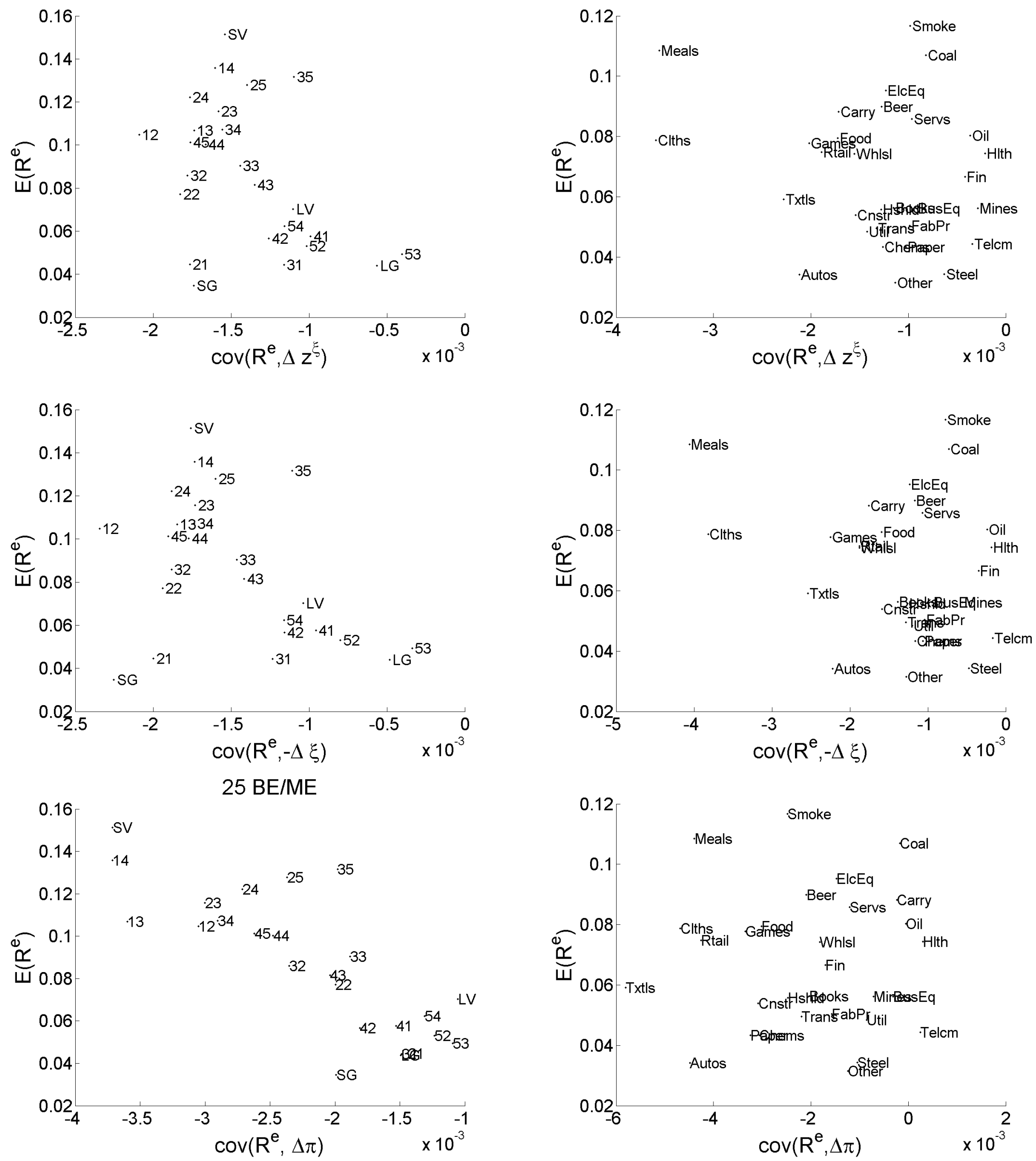


Figure 7 plots average returns versus covariances with the investment-specific shock (Z) and $\hat{\pi}$ for two sets of portfolios. I use two proxies for Z: first differences and innovations in the relative price of investment goods (ξ). The relative price of investment (ξ) comes from Israelsen (2010) who extends the quality-adjusted price series of Gordon (1990) to 2008. I compute innovations ξ_t^{ϵ} from $\ln \xi_t = a_0\, t + \rho \ln \xi_{t-1} + \xi_t^{\epsilon}$. I allow the time trend to vary before and after 1982 following the findings of Fisher (2006). I construct $\hat{\pi}$ using the full solution of the model and data consumption growth and the investment-output ratio (non-residential fixed investment divided by GDP excluding government spending). The left panel (A) uses the 25 portfolios sorted on market-equity and book-to-market from Fama and French (1993). The right panel (B) uses the 30 industry portfolios of Fama and French (1997). Sample includes annual data from 1963:2008.

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