Investment Shocks and Asset Prices*

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Abstract

I explore the implications for asset prices and macroeconomic dynamics of shocks that improve real investment opportunities and thus affect the representative household's marginal utility. These investment shocks generate differences in risk premia due to their heterogenous impact on firms: they benefit firms producing investment relative to firms producing consumption goods, and increase the value of growth opportunities relative to the value of existing assets. Using data on asset returns, I find that a positive investment shock leads to high marginal utility states. A general equilibrium model with investment shocks matches key features of macroeconomic quantities and asset prices.

1 Introduction

The second half of the twentieth century saw remarkable technological innovations, the majority of which took place in equipment and software. These technological innovations affect consumption only to the extent that they are implemented through the formation of new capital stock. The literature refers to these types of innovations as an investment shock, since they alter the real investment opportunity set in the economy. Investment shocks have proven quite successful in explaining long-run growth and have been adopted as a standard feature of many dynamic stochastic general equilibrium models.

The goal of this paper is to explore the implications of a standard real-business cycle model with investment shocks for asset prices and macroeconomic dynamics. I consider a

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general equilibrium model in which both consumption and asset prices are endogenously determined. In equilibrium, asset risk premia are determined by the covariance of asset returns with the growth rate of the representative household's marginal utility of consumption. Thus, in order to understand the effects of investment shocks for asset prices, it is necessary to determine how these shocks affect the representative household's marginal utility and to establish how investment shocks affect the cross-section of firms.

Investment shocks lower the resource cost of producing new capital goods, and thus alter the tradeoff between current and future consumption. In particular, a positive investment shock leads to lower consumption today, as the economy reallocates resources from the production of consumption goods to the production of investment goods. This increase in investment leads to higher capital stock in the future, which in turn increases future consumption. Hence, investment shocks affect the representative household's marginal utility of consumption, even though these shocks do not directly affect the production of consumption goods. Whether a positive investment shock increases or lowers the representative household's marginal utility of consumption depends on the specification of preferences. Under a time-separable utility specification, a positive investment shock leads to high marginal utility states, because households are willing to give up consumption today. More generally, the relation between the elasticity of intertemporal substitution and the coefficient of risk aversion determines whether a positive investment shock leads to high- or low-marginal utility states.

Investment shocks do not affect all firms equally in the cross-section. A positive investment shock benefits firms producing investment goods relative to firms producing consumption goods. Furthermore, a positive investment shock increases the value of investment opportunities relative to the value of existing assets in each sector. Hence, investment shocks affect firms differentially depending on whether they derive most of their value from their growth opportunities or assets in place.

Therefore, given that investment shocks affect the representative household's marginal utility, the model generates differences in risk premia between investment- and consumption-goods producers, as well as between firms with different opportunities to invest. The sign of these risk premia is informative as to whether a positive investment shock results in high or low marginal utility of wealth states. If a positive investment shock leads to states of the world where the marginal value of a dollar is high, assets which appreciate in relative terms following a positive investment shock should earn lower returns on average, and vice versa. In the data, investment firms, and firms with high growth opportunities earn lower returns on average than consumption firms and firms with low growth opportunities respectively. Thus, the data suggests that investment shocks lead to high marginal utility of wealth states, or equivalently, they carry a negative risk premium.

I calibrate the model to match the moments of real variables and asset returns. In my calibration, the elasticity of intertemporal substitution is lower than the reciprocal of risk aversion, which is a sufficient condition for the investment shock to carry a negative risk premium. Thus, the model provides a novel justification for the value premium puzzle. As long as growth firms derive most of their value from the present value of their growth opportunities and value firms derive most of their value from existing assets, growth firms will appreciate relative to value firms following a positive investment shock. Thus, growth firms are attractive to investors despite their low average returns, because they appreciate in value when real investment opportunities are high. In addition, the model matches the volatility and comovement of real economic variables, while generating realistic risk premia for asset returns.

I explore the implications of investment shocks for macroeconomic dynamics and asset risk premia in the data. The model suggests two ways of measuring investment-specific technological change in the data, each with its own relative advantages and disadvantages. The first measure is derived from data on the relative price of new equipment, as used in most of the literature on investment-specific shocks. The advantage of this proxy is that it is based on real economic data, but it is available only at low frequencies. The second measure is the relative stock returns of investment- to consumption-goods producers. This is a new measure of investment shocks, and has the advantage that it is available at high frequencies since it is based on financial data. My new measure generates responses in quantity variables that are consistent with their theoretical response to an investment shock. Indeed, I find that when firms producing investment goods realize higher stock returns relative to firms producing consumption goods, investment and hours increase, whereas both the relative price of investment and consumption fall.

The empirical evidence is consistent with my model, using either measure of investment shocks. I document that firms with high exposure to the investment shock tend to have lower returns on average than firms with low investment-shock exposures. The estimated risk premium associated with the investment shock is negative and statistically significant across specifications and across a variety of assets. In addition, I find that value and growth firms display differential sensitivity to the investment shock. This differential sensitivity to the investment shock, combined with the associated differences in asset premia, imply that my model improves upon the standard asset pricing models, namely the CAPM and Consumption CAPM, in explaining the cross-section of asset returns.

In summary, my results illustrate how the cross-section of asset returns can identify macroeconomic shocks. In addition, the cross-sectional moments of asset prices impose additional restrictions on the properties of shocks used in macroeconomic models, and thus can provide additional guidance in the search for the real cause of macroeconomic fluctuations.

2 Related literature

My paper fits into a growing literature that explores the implications of real business cycle models for asset prices [Jermann, 1998; Tallarini, 2000; Boldrin, Christiano and Fisher, 2001; Christiano and Fisher, 2003]. These studies find that, in economies with endogenous production, the equity premium and risk-free rate puzzles are exacerbated, because high risk aversion implies endogenously smooth consumption. Christiano and Fisher (2003) is the only other paper that explores the implications of investment shocks for asset prices, but it focuses on the price of equipment and the equity premium. In contrast to Christiano and Fisher (2003), I focus on the implications of investment shocks for the cross-section of risk premia. Gomes, Kogan and Yogo (2009) also explore the asset-return dynamics of firms producing different goods. Gomes et al. (2009) use a general equilibrium model with productivity shocks and study firms that produce durable and non-durable consumption goods. I use a model that has productivity shocks as well as investment shocks and study firms that produce consumption and investment goods.

Investment-specific technology shocks have become a standard feature of real business cycle models [Solow, 1960; Greenwood, Hercowitz and Krusell, 1997; Greenwood, Hercowitz and Krusell, 2000; Fisher, 2006]. Greenwood et al. (1997) use the price of new equipment as an empirical measure of the investment shock, and argue that such shocks are important determinants of long-run growth. Fisher (2006) reaches similar conclusions, by identifying the investment shock through long-run restrictions in a vector auto regression. I contribute to this literature by showing how financial data can be useful in measuring investment shocks.

Whether investment shocks are important drivers of business cycle fluctuations largely depends on their volatility. Fernandez-Villaverde and Rubio-Ramirez (2007), Justiniano and Primiceri (2008) and Justiniano, Primiceri and Tambalotti (2010) estimate large-scale macroeconomic models using maximum likelihood and Baysian methods, but reach different conclusions regarding the volatility of investment shocks. This difference in estimates arises because Fernandez-Villaverde and Rubio-Ramirez (2007) include the relative price of new equipment as an observable, whereas Justiniano and Primiceri (2008) and Justiniano et al. (2010) do not. Justiniano, Primiceri and Tambalotti (2011) reconciles the previous evidence by estimating a DSGE model with frictions, where the investment shock is decomposed into two separate shocks. The first shock affects the marginal rate of transformation of consumption into investment goods and is identified using the price of new equipment. The second shock affects the rate of transformation of investment goods into installed capital. Justiniano et al. (2011) find that the second investment shock plays an important role in explaining business cycle fluctuations, whereas the first investment shock plays a minor role. I adopt the two-shock specification in Justiniano et al. (2011) and show that it helps reconcile the low volatility of equipment price with the high volatility of asset returns.

Finally, my paper is connected to the investment-based asset pricing literature [Cochrane, 1996]. This literature assumes a specific form for the stochastic discount factor, and then derives and tests implications for asset returns and investment decisions. Cochrane (1996) tests a factor model for asset returns, assuming that the stochastic discount factor is linear in physical investment returns. These investment returns are approximately equal to investment growth. The empirical estimates of Cochrane (1996) imply that states with positive non-residential investment growth are states where the marginal utility of wealth is high. My model provides a theoretical justification for this empirical finding, since investment growth is driven by innovations to the investment shock.

3 General equilibrium model

In this section, I build a general equilibrium model to investigate the link between investment-specific shocks and asset prices. I cast the model in continuous-time, which can be viewed as the limit $\Delta t \to 0$ of a discrete-time, locally-quadratic approximation. Relative to a discrete-time model, the continuous-time formulation allows me to derive cleaner expressions for asset risk premia, and also offers advantages when computing numerical solutions for macroeconomic quantities and asset prices. These numerical solutions are more accurate than log-linear approximations around the steady state, especially for asset risk premia.

3.1 Households

There exists a continuum of identical households which maximize a utility index J_0, over sequences of consumption C_t, and leisure N_t. I specify preferences as in Duffie and Epstein (1992), which are the continuous time analog of Epstein and Zin (1989). Utility is defined recursively as follows:

$$J_0 = E_0 \int_0^\infty h(C_t, N_t, J_t)dt, \tag{1}$$

where

$$h(C,N,J) = \frac{\rho}{1-\theta^{-1}} \left(\frac{(CN^\psi)^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)\, J \right). \tag{2}$$

In this formulation, ρ is the time-preference parameter, γ is the coefficient of relative risk aversion, θ is the elasticity of intertemporal substitution (EIS), and ψ affects the relative shares of consumption and leisure. I require that $\psi(1-\theta^{-1}) < 0$ in order to ensure that leisure is a good. In the case where the coefficient of risk aversion equals the reciprocal of the EIS ($\gamma = \theta^{-1}$), these preferences become separable over time and reduce to a special case of the preferences studied by King, Plosser and Rebelo (1988):

$$J_0 = E_0 \int_0^{\infty} e^{-\rho(s-t)} \frac{\left(C_t N_t^{\psi}\right)^{1-\gamma}}{1-\gamma} dt. \tag{3}$$

Households supply $1 - N$ units of labor that can be freely allocated between the two sectors,

$$L_{C,t} + L_{I,t} = 1 - N_t. \tag{4}$$

3.2 Firms and technology

In this economy, production takes place in two separate sectors: a sector producing the consumption good, and a sector producing the investment good.

Consumption sector

The consumption goods sector produces the consumption good C with two factors of production, sector specific capital K_C and labor L_C, according to the following Cobb-Douglas technology:

$$C_t = A_t \, K_{C,t}^{\beta_C} \, L_{C,t}^{1-\beta_C}, \tag{5}$$

Output in the consumption sector is subject to a disembodied productivity shock A that evolves according to a geometric random walk with growth rate μ_A and volatility σ_A:

$$dA_t = \mu_A \, A_t \, dt + \sigma_A \, A_t \, dB_t^A. \tag{6}$$

Innovations in the consumption productivity shock A are driven by the Wiener process B_t^A. The Wiener process B_t^A is a stochastic process with continuous paths and normally distributed increments $B_s^A - B_t^A \sim \mathcal{N}(0, s-t)$.

Consumption producers can purchase the investment good and increase their capital stock at a rate i_C. The capital stock K_C depreciates at a rate δ, and thus its law of motion is given by:

$$dK_{C,t} = \left(i_{C,t} - \delta\right) \, K_{C,t} \, dt. \tag{7}$$

Firms can increase their capital stock by an absolute amount $i_C\, K_C$ by purchasing $Z_m^{-1} c(i_C) K_C$ units of the investment good at a relative price p^I. The total investment cost, measured in units of the investment good, is comprised of two parts. The first component Z_m^{-1} is stochastic, while the second component $c(i_C)K_C$ is an increasing and convex function of the investment rate i_C.

A positive shock to the marginal efficiency of investment Z_m implies that investment goods can be transformed into capital in a more efficient way, as in Justiniano et al. (2011). The shock Z_m follows a random walk with volatility $\sigma_{Z,m}$:

$$dZ_{m,t} = \sigma_{Z,m}\, Z_{m,t}\, dB_t^{Z,m}, \tag{8}$$

where $B_t^{Z,m}$ is a Wiener process, whose increments are independent of dB_t^A. The shock Z_m can be interpreted as either an improvement in the quality of investment goods, i.e. new investment goods can be transformed into more effective capital units, or as additional frictions to the process through which investment goods are turned into capital ready for production. As argued by Justiniano et al. (2011), these frictions could be related to the efficiency of the financial sector, and can be interpreted as a reduced form of the financial accelerator model of Carlstrom and Fuerst (1997).

The second component of the total investment cost $c(i_C)K_C$ is given by:

$$c(i_C)K_C = \left(\frac{1}{\lambda}\left(1+i_C\right)^{\lambda} - \frac{1}{\lambda}\right) K_C. \tag{9}$$

Here, $\lambda \geq 1$ captures the degree of the adjustment costs.[1]

In each period, firms hire labor at a competitive wage w and purchase new investment goods to maximize their value:

$$S_{C,t} = E_t \int_t^{\infty} \frac{\pi_s}{\pi_t} \left(A_s K_{C,s}^{\beta_C}\, L_{C,s}^{1-\beta_C} - w_s L_{C,s} - p_s^I\, Z_m^{-1}\, c\left(i_{C,s}\right) K_{C,s} \right) ds, \tag{10}$$

where π is the Lagrange multiplier of the social planner's problem on the resource constraint (5) and represents the price of consumption in each state, normalized in units of probability.

[1] My specification of the total investment cost in equation (9) implies adjustments costs equal to $(1 + i_C)^{\lambda}/\lambda - 1/\lambda - i_C$. In the case of $\lambda = 1$ it reduces to $c(i_C) = i_C$. For $\lambda = 2$, it specializes to the case of quadratic adjustment costs studied in the literature, $c(i_C) = i_C + i_C^2/2$. Since $c'(0) = 1$, adjustment costs are zero at $i_C = 0$. Finally, $c^{-1}(i_C)$ gives the rate of transformation of investment goods to installed capital.

Investment sector

The investment goods sector produces Y_I units of the investment good using sector-specific land K_I and labor L_I:

$$Y_I = Z_{I,t}\, K_I^{\beta_I}\, L_{I,t}^{1-\beta_I}. \tag{11}$$

Here, Z_I is a shock to total factor productivity in the investment sector, and follows a geometric random walk with growth rate μ_Z and volatility $\sigma_{Z,I}$:

$$dZ_{I,t} = \mu_Z\, Z_{I,t}\, dt + \sigma_{Z,I}\, Z_{I,t}\, dB_t^{Z,I}, \tag{12}$$

where $B_t^{Z,I}$ is a Wiener process, whose increments are independent of dB_t^A and $dB_t^{Z,m}$. A positive shock to Z_I increases the productivity of the investment sector. In this case, the economy can produce the same amount of new investment goods using fewer labor resources L_I.

Firms in the investment sector represent claims on the land K_I used to produce investment goods. Investment firms sell their output at a competitive price p^I and hire labor at a price w to maximize their market value:

$$S_{I,t} = E_t \int_t^\infty \frac{\pi_s}{\pi_t}\, \left(p_s^I\, Z_{I,s}\, K_I^{\beta_I}\, L_{I,s}^{1-\beta_I} - w_s\, L_{I,s}\right)\, ds. \tag{13}$$

The assumption that K_I is fixed reduces the number of state variables and thus greatly simplifies the model. Without loss of generality I fix the amount of land at $K_I = 1$. I extend the model to allow for endogenous capital accumulation in the investment sector in Section 4.5.

3.3 Competitive equilibrium

Social planner's problem

The model features three sources of uncertainty: i) total factor productivity shocks in the consumption sector, denoted by A; ii) total factor productivity shocks in the investment sector, denoted by Z_I; iii) shocks to the marginal efficiency of investment, denoted by Z_m. Since both Z_I and Z_m affect the tradeoff between consumption and capital, I refer to both as the investment shock. Irrespective of whether these shocks capture a real technological disturbance or a variation in the efficiency of the financial sector in financing new loans, both play a similar role in the model. Thus, I will refer to the composite shock $Z \equiv Z_I \cdot Z_m$

as the investment shock, which evolves according to:

$$dZ_t = \mu_Z \, Z_t \, dt + \sigma_Z \, Z_t \, dB_t^Z, \tag{14}$$

where $\sigma_Z^2 = \sigma_{Z,I}^2 + \sigma_{Z,m}^2$ and $dB_t^Z = \sigma_{Z,I}/\sigma_Z \, dB_t^{Z,I} + \sigma_{Z,m}/\sigma_Z \, dB_t^{Z,m}$.

Given that the welfare theorems hold, it is sufficient to solve the social planner's problem. The planner's value function, J, satisfies the Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} 0 \;\; = \;\; & \max_{L_I, L_C, i_C, N} \Big\{ h(C,N,J) + (i_C - \delta) J_{K_C} K_C + \mu_A \, J_A \, A + \frac{1}{2} \, \sigma_A^2 \, J_{AA} \, A^2 \\ & + \mu_Z \, J_Z \, Z + \frac{1}{2} \sigma_Z^2 \, J_{ZZ} \, Z^2 \Big\}, \end{aligned} \tag{15}$$

The social planner's value function takes the form

$$J(A, Z, K_C) = \frac{(A \, K_C^{\beta_C})^{1-\gamma}}{1-\gamma} f(\omega), \tag{16}$$

where $f(\omega)$ satisfies an ordinal differential equation (see Appendix A), and $\omega \equiv \ln(Z/K_C)$.

The current state of the economy can be summarized by the variable ω. The state variable ω captures the real investment opportunities in the economy, which depend on the deviation of capital K_C from an optimal level determined by the investment shock Z. The endogenous variable ω evolves according to the following stochastic differential equation:

$$d\omega_t = \left(\mu_Z + \delta - \frac{1}{2} \sigma_Z^2 - i_C(\omega) \right) \, dt + \sigma_Z \, dB_t^Z. \tag{17}$$

The expected growth rate of ω is decreasing in ω because $i_C'(\omega) > 0$. Thus, a sufficient condition for ω to have a stationary distribution is that $\mu_Z + \delta - \sigma_Z^2/2 > 0$.

The stationary variable ω captures the fluctuations in economic quantities around a common stochastic trend, which depends on the productivity shock A and the investment shock Z. Specifically, the value of consumption, output and investment, all measured in consumption units, share the stochastic trend $\Xi = AZ^{\beta_C}$, and grow at the average rate $\mu_A + \beta_C \mu_Z$. Thus, along a balanced growth path, C/Ξ and I/Ξ are stationary and only depend on ω.

Household first order conditions

Households purchase state-contingent consumption at a price π and supply labor in a competitive market at a price w. Their first order condition with respect to consumption is:

$$\pi_t = \exp\left(\int_0^t h_J(C_s, N_s, J_s)\, ds\right) h_C(C_t, N_t, J_t). \tag{18}$$

Here, π_t is the Lagrange multiplier on the cental planner's problem on the resource constraint given by (5). The exponential term captures the effect of discounting and is reduced to $\exp(-\rho t)$ in the case where preferences are time-separable ($\gamma = \theta^{-1}$). The second term $h_C(C, N, J)$ denotes the exchange rate between utils and consumption in each state, and corresponds to marginal utility in the time-separable case.

The households' labor supply decision is intra-temporal and depends only on current consumption C_t and the wage, w_t:

$$\frac{h_N(C_t, N_t, J_t)}{h_C(C_t, N_t, J_t)} = w_t \Rightarrow N_t = \psi\, \frac{C_t}{w_t}. \tag{19}$$

As equation (19) shows, the specification of preferences (2) implies that the income elasticity of labor supply $d\log(1-N)/d\log C$ is equal to the opposite of the Frisch elasticity of labor supply $d\log(1-N)/d\log w$, as in King et al. (1988).

Firm first order conditions

Firms in the consumption sector decide how much to invest in new capital i_C by purchasing the investment good at a price p^I. Their first order condition is inter-temporal:

$$p_t^I\, Z_{m,t}^{-1}\, c'(i_{C,t}) = \frac{J_{K_{C,t}}}{h_{C,t}}, \tag{20}$$

where J_{K_C}/h_C is the marginal value of installed capital in the consumption sector, measured in consumption units. Equation (20) states that the relative price of uninstalled capital p^I times the marginal installation cost equals the marginal value of capital in the consumption sector.

In my calibration, I find that the price of investment goods p^I is declining with respect to the composite investment shock Z.[2] This suggests using innovations in the price of new

[2] A positive productivity shock in the investment sector Z_I increases the supply of investment goods, thus their relative price p^I falls. By contrast, the shock to the marginal efficiency of investment Z_m has a theoretically ambiguous effect on the price of investment goods p^I. An increase in Z_m has a direct positive effect on p^I because it allows transforming a given amount of investment goods into more units of installed

equipment as the first natural empirical proxy for investment-specific shocks. By contrast, a positive productivity shock in the consumption sector A increases the demand for new capital goods, leading to an increase in the price of investment goods p^I.

Consumption and investment firms hire labor L_C and L_I at the competitive rate w. Thus, the first order condition for the consumption firms is:

$$(1-\beta_C)\, A_t K_{C,t}^{\beta_C} L_{C,t}^{-\beta_C} = w_t, \tag{21}$$

and for the investment firms is:

$$(1-\beta_I)\, p_t^I Z_t K_I^{\beta_I} L_{I,t}^{-\beta_I} = w_t. \tag{22}$$

In equilibrium, the marginal product of labor in both sectors must be equal, which we can see from the two first-order conditions (21)-(22). Combining labor market clearing (4) with the first-order conditions (19) and (21), we obtain that household leisure N and the allocation of labor to the consumption sector L_C are given by:

$$N_t = \frac{\psi}{1-\beta_C+\psi}(1-L_{I,t}) \tag{23}$$

$$L_{C,t} = \frac{1-\beta_C}{1-\beta_C+\psi}(1-L_{I,t}). \tag{24}$$

Given the central planner's value function (16) and the equations characterizing labor market equilibrium (20)-(24), it is important to note that labor decisions are independent of the productivity shock in the consumption sector A. The intuition is that the productivity shock in the consumption sector has both an income and a substitution effect on labor supply. However, these two effects exactly offset each other, due to the specification of preferences in (2), and consequently there is no effect on total labor supply in this model.

By contrast, labor supply does respond to the investment shock Z. A positive investment shock Z increases the marginal product of labor in the investment sector, and hence the equilibrium wage. In response to this increase in equilibrium wage, households increase their supply of labor and substitute labor away from producing consumption goods and into producing investment goods. In equilibrium, a positive investment shock induces an increase in investment and a decline in consumption and leisure. However, this fall in consumption is temporary. The increase in capital accumulation in the consumption sector implies that

capital, thereby increasing the demand for investment goods. However, an increase in Z_m has also an indirect negative effect on p^I because it increases the equilibrium investment rate and therefore leads to higher marginal adjustment costs $c'(i_C)$. In my calibration, I find that the two effects cancel out and the price of equipment is fairly insensitive to Z_m.

consumption growth will be higher in the future.

Asset prices

In equilibrium, asset risk premia are determined by the covariance of asset returns with the stochastic discount factor. The stochastic discount factor is defined as the growth rate of the Lagrange multiplier π of the cental planner's problem on the resource constraint (5). The multiplier π denotes the price of consumption in each state, normalized by probability, and is also equal to the marginal utility of wealth. Assets that tend to pay off when the state price of consumption is high are more desirable and thus command lower risk premia. In particular, for any asset i with price S_i and dividend stream D_i, the total expected return from holding that asset in excess of the risk free rate equals:

$$E_t\left[\frac{d\,S_{i,t}+D_{i,t}\,dt}{S_{i,t}}-r_{f,t}\,dt\right]=-cov_t\left[\frac{d\,\pi_t}{\pi_t},\frac{d\,S_{i,t}}{S_{i,t}}\right]. \tag{25}$$

The stochastic discount factor is correlated with the two sources of uncertainty in the model: the consumption productivity shock A, and the investment shock Z. An application of Ito's lemma to equation (18) implies that the stochastic discount factor equals:

$$\frac{d\pi_t}{\pi_t}=-r_{f,t}\,dt-b_A\,dB_t^A-b_Z(\omega_t)\,dB_t^Z\,, \tag{26}$$

where $b_A=\gamma\sigma_A$ and

$$b_Z(\omega)=-\left(\left(\theta^{-1}(1-\beta_C)+\psi(\theta^{-1}-1)\right)\frac{L_I'(\omega)}{1-L_I(\omega)}+\left(\theta^{-1}-\gamma\right)\frac{f'(\omega)}{(1-\gamma)f(\omega)}\right)\sigma_Z. \tag{27}$$

Equations (25)-(26) shows that asset risk premia are determined by the exposure to the two innovations in A and Z

$$E_t\left[\frac{d\,S_{i,t}+D_{i,t}\,dt}{S_{i,t}}-r_{f,t}\,dt\right]=b_A\,cov_t\left(\frac{d\,S_{i,t}}{S_{i,t}},dB_t^A\right)+b_Z(\omega_t)\,cov_t\left(\frac{d\,S_{i,t}}{S_{i,t}},dB_t^Z\right). \tag{28}$$

Asset risk premia due to either technology shock can be decomposed into the product of the price of risk (b_A or b_Z) times the asset's risk exposure, as measured by the covariance of an asset's return with this shock. Here, I define the price of risk of a random variable $\tilde{y}$ as the Sharpe ratio of a security whose payoff is perfectly correlated with realizations of $\tilde{y}$. The price of risk associated with $\tilde{y}$ depends on how the state price of consumption π is correlated with the shock $\tilde{y}$. If the marginal utility of wealth, and hence the state price of consumption, is lower following an increase in $\tilde{y}$, then this shock will carry a positive risk premium. Since households attach a lower value to these states, they are willing to pay a lower price for a security that pays off in low consumption states, or equivalently they demand a positive risk premium. Conversely, if the state price of consumption is higher following a positive shock to $\tilde{y}$, households are willing to pay a higher price for securities that are positively correlated with $\tilde{y}$, and thus the risk premium is negative.

The price of risk of the productivity shock in the consumption sector b_A is constant and depends only the risk aversion coefficient γ and the volatility of the consumption productivity shock A. The fact that the price of risk b_A is positive implies that households demand a positive risk premium to invest in securities that are positively correlated with the consumption productivity shock A.

Examining equation (27) more closely, the risk premium of the investment shock b_Z is comprised of two parts, each capturing concerns about current and future consumption respectively. The first term, which reflects concerns about current consumption, is negative because both consumption and leisure temporarily fall after a positive investment shock. This temporary decline in consumption and leisure increases the price of consumption in that state and the magnitude of this premium depends on the willingness of households to substitute consumption across time (EIS).

The second term in equation (27) captures concerns about future consumption. The sign of this term is ambiguous and depends on the elasticity of intertemporal substitution θ versus the coefficient of risk aversion γ.[3] A positive investment shock increases future consumption growth through an increase in the allocation of labor to the investment sector which leads to an increase in the capital stock. If households have preferences for later resolution of uncertainty ($\gamma\theta < 1$), the second term in equation (27) will also be negative. In this case, a positive investment shock leads to high marginal utility of wealth states. Conversely, if households have preferences for early resolution of uncertainty ($\gamma\theta > 1$), the second term in (27) will be positive. In this case, whether a positive investment shock leads to high marginal utility of wealth states depends on whether the first or second term in (27) dominates.

In summary, a positive investment shock leads to a U-shaped response in consumption growth: it initially falls because the economy allocates more resources to the investment-goods sector in order to take advantage of the improvement in technology. However, as the new technology starts bearing fruit, the growth rate of consumption increases. If the EIS and the coefficient of risk aversion are both sufficiently low, states characterized by good real investment opportunities will also be states of high marginal value of wealth. The

[3] The term $\frac{f'(\omega_t)}{(1-\gamma)f(\omega_t)}$ is greater than zero because the marginal product of capital J_{K_C} is positive.

intuition is that a low value of the EIS and risk aversion implies that households would like to smooth consumption paths across time rather than across states, and thus value that extra unit of wealth more when they face an endogenously steep consumption profile. Conversely, when the EIS and risk aversion are both high, investors are relatively unconcerned about smoothing over time, but are worried about smoothing across states. A positive investment shock then represents a state of the world where the marginal value of consumption is low, since in those states their continuation utility is high.

Investment and consumption firms

The next step in characterizing asset risk premia is to analyze the sensitivity of asset values to the technology shocks A and Z. The value of the investment-goods S_t^I and the consumption-goods sector S_t^C solve the following stochastic differential equations:

$$\frac{dS_t^I + D_{I,t}\,dt}{S_t^I} = E_t\left[\frac{dS_t^I + D_{I,t}\,dt}{S_t^I}\right] + \sigma_x dB_t^A + \zeta_I(\omega)\,\sigma_Z\,dB_t^Z \tag{29}$$

$$\frac{dS_t^C + D_{C,t}\,dt}{S_t^C} = E_t\left[\frac{dS_t^C + D_{C,t}\,dt}{S_t^C}\right] + \sigma_x dB_t^A + \zeta_C(\omega)\,\sigma_Z\,dB_t^Z \tag{30}$$

where the sensitivities of each sector to the investment shock $\zeta_I(\omega)$ and $\zeta_C(\omega)$ are defined in appendix A.

Following a positive investment shock Z, the investment sector appreciates in value relative to the consumption sector, that is $\zeta_I(\omega) > \zeta_C(\omega)$. By contrast, a positive consumption productivity shock A affects both sectors symmetrically. A positive shock to A increases the productivity of the consumption sector, leading to an increase in the demand for new capital goods and hence their price. Since the supply of capital goods remains unchanged, the price of investment goods p^I and the profits of the investment firms increase by the same proportion as the productivity of the consumption sector. Thus, the stock returns of both sectors have the same exposure to the consumption productivity shock.

Equations (29)-(30) show that a portfolio long investment and short (minus) consumption producers (IMC) is perfectly correlated with innovations in the investment shock Z. Thus, realized returns on the IMC portfolio can serve as a second empirical proxy for Z, illustrating how the cross-section of stock returns can be an additional source of information for real economic shocks. From the return dynamics of both sectors given by equations (29)-(30) and the equation characterizing the risk-return tradeoff (28), it follows that the difference in risk premia between investment- and consumption-firms is described by:

$$E_t\left[\frac{d\,S_{I,t} + D_{I,t}\,dt}{S_{I,t}} - \frac{d\,S_{C,t} + D_{C,t}\,dt}{S_{C,t}}\right] = \Big(\zeta_I(\omega) - \zeta_C(\omega)\Big)\, b_Z(\omega_t)\,\sigma_Z\,dt. \tag{31}$$

Investment-firms earn lower average returns than consumption-firms if the price of risk for the investment shock is negative ($b_Z < 0$). Thus, differences in average returns between investment and consumption firms are informative about the risk premium associated with the investment shock.

Market portfolio

The value of the market portfolio equals the sum of the values of the investment- and consumption-sectors. The total return from holding the market portfolio is given by:

$$\frac{dS_t^M + D_{M,t}\,dt}{S_t^M} = E_t\left[\frac{dS_t^M + D_{M,t}\,dt}{S_t^M}\right] + \sigma_x dB_t^A + \zeta_M(\omega)\,\sigma_Z\,dB_t^Z, \tag{32}$$

where the sensitivity of the market portfolio to the investment shock $\zeta_M(\omega)$ is defined in the appendix. Combining the return dynamics of the market portfolio (32) with the characterization of risk premia (28), the equity premium implied by the model equals:

$$E_t\left[\frac{d\,S_{M,t} + D_{M,t}\,dt}{S_{M,t}} - r_{f,t}\,dt\right] = \left[\gamma\sigma_A^2 + \zeta_M(\omega)b_Z(\omega_t)\,\sigma_Z\right]dt. \tag{33}$$

The equity premium is comprised of two terms. The first term $\gamma\sigma_A^2$ in equation (33) indicates the compensation to the holders of the market portfolio for risk due to the productivity shock in the consumption sector. Without the investment shock ($\sigma_Z = 0$), this is the only component of the equity premium and the equity premium puzzle arises. In this case, the model needs a risk aversion coefficient greater than 100 to simultaneously match the equity premium (4.9%) and the low volatility of consumption growth (2%).

The second term $\zeta_M(\omega)b_Z(\omega_t)\,\sigma_Z$ in the equation characterizing the equity premium (33) reflects the compensation to investors for bearing risk due to the investment shock. Investors in the market portfolio are exposed to risk because a positive investment shock leads to an increase in both future dividends and the interest rate. When the value of the EIS is less than one, the increase in the risk-free rate dominates the effect of higher future dividends. In this case, the value of the stock market falls, implying $\zeta_M(\omega) < 0$. If, in addition, the risk premium on the investment shock is negative ($b_Z(\omega_t) < 0$), the equity premium is higher.

Value and growth

The finance literature has extensively documented the value premium puzzle, namely the finding that firms with high book-equity to market-equity ratios (book-to-market) have substantially higher average returns than firms with low book-to-market [Fama and French, 1993]. This difference in average returns is economically large, since it is close in magnitude to the equity premium. The book-to-market ratio is closely related to the inverse of Tobin's Q, as it compares the replacement cost of the firm's assets to their market value.[4] Thus, firms with high book-to-market are often referred to as value firms, while firms with low book-to-market are referred to as growth firms.

The puzzle arises because value and growth firms appear to have the same systematic risk, measured by their exposure to the market portfolio. Furthermore, Fama and French (1993) show that a portfolio of value minus growth firms is a separate risk factor in the time-series of returns, in addition to the market portfolio. In this section, I argue that a model with investment shocks can rationalize why households are willing to invest in growth firms despite earning lower risk premia. In addition, the model sheds light on what makes value firms riskier than growth firms, and the type of risk that they are exposed to.

The value premium puzzle is mainly a within- rather than between- industry phenomenon [Cohen, Polk and Vuolteenaho, 2003]. The model described thus far cannot explain this phenomenon because it only considers one representative firm for each sector in the economy. Thus, to create the analog of value and growth firms within each sector, I consider two firms in each sector: one that consists purely of assets in place, and the other that consists purely of future growth opportunities. In this case, I restrict attention to the consumption-goods sector, since the capital stock in the investment-goods sector K_I is fixed. When I extend the model to allow for endogenous capital accumulation in the investment sector, I find that the same conclusions also apply to firms in the investment (see supplemental Appendix).

The value of assets in place in the consumption sector is given by the value of all cashflows accruing from existing assets:

$$S_t^{VC} = \max_{L_{C,s}} E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(A_s (K_{C_t} e^{-\delta(s-t)})^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} \right) ds. \tag{34}$$

In the absence of arbitrage, the value of growth opportunities must equal the residual

[4] The difference arises because i) firms are also financed by debt; ii) the denominator in measures of Tobin's Q is the replacement cost of capital rather than the book-value of assets. Nevertheless, Tobin's Q and book-to-market generate very similar dispersion in risk premia.

value given by the difference between S^{C} and S^{VC}:

$$S_t^{GC} = S_t^{C} - S_t^{VC}. \tag{35}$$

This residual value is positive because it represents the present value of rents to consumption firms. Consumption firms earn rents because the value of installed and uninstalled capital differ due to the presence of adjustment costs to capital, or equivalently Tobin's Q is greater than one.

An application of Ito's lemma to the solutions of equations (34)-(35) implies that the value of growth opportunities S^{GC} and the value of assets in place S^{VC} in the consumption sector satisfy the following stochastic differential equations:

$$\frac{d\,S_t^{GC} + D_t^{GC}\,dt}{S_t^{GC}} = E_t\left[\frac{d\,S_t^{GC} + D_t^{GC}\,dt}{S_t^{GC}}\right] + \sigma_x dB_t^A + \zeta_G(\omega)\,\sigma_Z\,dB_t^Z \tag{36}$$

$$\frac{d\,S_t^{VC} + D_t^{VC}\,dt}{S_t^{VC}} = E_t\left[\frac{d\,S_t^{VC} + D_t^{VC}\,dt}{S_t^{VC}}\right] + \sigma_x dB_t^A + \zeta_V(\omega)\,\sigma_Z\,dB_t^Z, \tag{37}$$

where the investment-shock sensitivities $\zeta_G(\omega)$ and $\zeta_V(\omega)$ are defined in Appendix A.

I find that the relative value of growth opportunities to assets in place (S^{GC}/S^{VC}) is an increasing function of ω, implying that $\zeta_G(\omega) > \zeta_V(\omega)$. Using equations (36)-(37) and the characterization of risk premia (25), the difference in risk premia between the growth and value firms in the consumption sector equals

$$E_t\left[\frac{d\,S_t^{GC} + D_t^{GC}\,dt}{S_t^{GC}} - \frac{d\,S_t^{VC} + D_t^{VC}\,dt}{S_t^{VC}}\right] = \Big(\zeta_G(\omega) - \zeta_V(\omega)\Big) b_Z(\omega_t)\,\sigma_Z\,dt$$

Value firms will earn higher risk premia than growth firms as long as the price of risk for the investment shock is negative ($b_Z < 0$). In this case, households are willing to invest in growth firms because their price appreciates relative to value firms in states of the world where the marginal value of consumption π is high. Thus, differences in average returns between value and growth firms in each sector contain information about the risk premium associated with the investment shock, in addition to that provided by the average returns between investment and consumption firms.

In addition, equations (36)-(37) show that the cross-section of stock returns has a two factor structure. All firms have the same exposure to the consumption productivity shock A, but they differ in their exposure to the investment shock depending on the fraction of firm value that is due to growth opportunities. The return spread between the value and growth firms is driven only by the investment shock Z, thus my model naturally generates a value factor, explaining the pattern of comovement documented by Fama and French (1993).

The existence of a second aggregate source of risk differentiates my paper from existing models with only one aggregate shock [Gomes, Kogan and Zhang, 2003; Zhang, 2005; Gala, 2010]. These models generate a value premium through a time-varying exposure to a single source of uncertainty, which is captured by the return to the market portfolio. Both explanations have similar predictions regarding risk premia. However, these models lead to very different implications about the main source of risk that investors are exposed to when buying value versus growth firms. In existing models, the value premium arises because value firms become riskier than growth firms in recessions. By contrast, here the value premium arises because the price of growth firms rises relative to value firms when real investment opportunities improve, as captured by a positive innovation to the investment shock Z.

If the market portfolio represents the only source of systematic risk that drives the returns to value and growth firms, once that risk is partialed out there should be no remaining source of comovement. To show that this is not the case, I perform a simple empirical exercise. I consider 10 portfolios of firms sorted on book-to-market, and compute the residuals from a regression on the market portfolio. I use weekly data on stock returns, so I allow the exposure to the market portfolio to vary every year. Even though this procedure eliminates the time-varying exposure of these portfolios to the market, the resulting residuals display a strong factor structure. The first principal component explains 60% of the variation in these residual returns, suggesting that value and growth firms appear to be exposed to a source of risk that is not fully captured by the market portfolio.

The data indicates the presence of at least two separate sources of risk in the cross-section of stock returns. Makarov and Papanikolaou (2009) provide evidence consistent with the view that the investment shock is one of these sources of risk, by analyzing the common factors in the cross-section of industry portfolios. One of the extracted factors has a correlation of 87% with the relative returns of investment- to consumption-good producers, which as we can see in equations (29)-(30) is perfectly correlated by the investment shock. I revisit this issue in Section 5.2, where I explore whether value and growth firms have differential exposure to measures of the investment shock.

4 Calibration

In this section, I explore whether a real business cycle model with investment shocks can simultaneously match the key moments of real economic variables and asset returns. This exercise reveals the quantitative importance of investment shocks as drivers of business cycle fluctuations and determinants of asset risk premia.

4.1 Parameter choice

The key parameter that affects the importance of investment shocks in the model is their volatility. In my benchmark calibration, I choose the volatility of the productivity shock in the investment sector $\sigma_{Z,I} = 3.5\%$ to match the volatility of the relative price of investment goods. I use $\sigma_{Z,m} = 13.5\%$ for the volatility of the shock to the marginal efficiency of investment, which lies within the confidence intervals estimated in Justiniano et al. (2011).

To illustrate the role of the shock to the marginal efficiency of investment, Z_m, I present a series of calibrations with $\sigma_{Z,m} = 0$. I consider three values of the productivity shock in the investment sector. In the first scenario, I choose a conservative value of $\sigma_{Z,I} = 4.5\%$ based on the estimates of Fernandez-Villaverde and Rubio-Ramirez (2007). Second, I chose an intermediate value of $\sigma_{Z,I} = 10\%$ based on the findings of Justiniano and Primiceri (2008) and Justiniano et al. (2010).[5] Finally, I choose a high value $\sigma_{Z,I} = 20\%$ to match the volatility of investment growth and of the relative returns of investment and consumption producers.

The rest of the parameters governing the dynamics of technology shocks are fairly standard. I calibrate the volatility of productivity shocks $\sigma_A = 2\%$ to match the volatility of consumption and the mean growth rate of the consumption sector $\mu_A = 0.1\%$ and the investment sector $\mu_Z = 4\%$ to simultaneously match the average growth of consumption, investment and output, the growth rate of relative prices of investment goods and the level of the risk free rate.

To calibrate household preferences, I choose a value for the elasticity of intertemporal substitution $\theta = 0.3$ consistent with Vissing-Jorgensen (2002), who estimates an EIS of 0.3 to 0.4 for stockholders using micro-level data. This value for the EIS also falls within the estimates reported by Hall (1988). Consistent with King et al. (1988), I set the utility share of labor to $\psi = 3$, to ensure that the steady-state level of hours worked approximately matches the US data.[6] I choose a value for the coefficient of risk aversion $\gamma = 1.1$ that is consistent with the real business cycle literature (e.g. King and Rebelo (1999)) and a low value for the subjective discount rate $\rho = 0.001$ to help the model match the low level of the risk-free rate.

On the production side, I choose the shares of labor equal to $\beta_I = 0.1$ and $\beta_C = 0.3$,

[5] The difference in estimates between these two sets of papers is mostly because Fernandez-Villaverde and Rubio-Ramirez (2007) include the relative price of new equipment as an observable variable, whereas Justiniano and Primiceri (2008) and Justiniano et al. (2010) do not.

[6] A parametrization $\psi = 3$ implies that the income elasticity of labor equals approximately -3, whereas the wage elasticity equals 3. These labor elasticities are in line with most macro models in the literature [King and Rebelo, 1999].

to help match the relative size of the two sectors (27% in the model vs 23% in the data), while generating a labor share of output of approximately 75%. Choosing $\beta_I < \beta_C$ is also motivated by the finding of Cummins and Violante (2002) that the investment sector is more labor intensive than the consumption sector. I select a depreciation rate of $\delta = 8.5\%$ to yield an investment-to-capital ratio of around 12% and a ratio of investment to consumption expenditures of around 24%, which are comparable to the data (11% and 21% respectively). I set the adjustment cost parameter equal to $\lambda = 1.15$, which is close to the no-adjustment cost case ($\lambda = 1$). This choice of λ implies that investment is very elastic with respect to Tobin's Q (elasticity of around 50), and that the average fraction of adjustment costs to total investment expenditures is less than 1%. This conservative value for λ ensures that investment growth is sufficiently volatile.[7]

Finally, the model features no debt, whereas in the data firms are financed approximately 40% by debt and 60% by equity. To facilitate better comparison between the data and the model, I multiply risk premia and standard deviations of stock returns by a number equal to 5/3 as in Boldrin et al. (2001) to better approximate the moments of levered claims on capital.

4.2 Numerical solution

I solve the social planner's value function $f(\omega)$ through value function and policy iteration following Kushner and Dupuis (1992). I simulate the model at a monthly frequency and aggregate the data to form quarterly or annual observations. I simulate 10,000 paths, each with a length of 100 years. For each sample, I drop the first half to remove the dependence on initial values. Figure 1 plots the solution of the model as a function of the state variable ω, along with its invariant distribution. These graphs summarize the three main intuitions of the model.

First, holding A and K_C fixed, consumption is a declining function of ω, while investment and output are increasing. The decline in consumption implies that the state price of consumption per unit of probability, which is equal to the marginal utility of wealth, is an increasing function of ω. Consequently, in my calibration the risk-premium associated with the investment shock (b_Z) is negative.

Second, the relative value of the investment to the consumption sector is an increasing function of ω. Holding Z_m constant, the relative price of investment goods p^I is decreasing in ω. Consequently, innovations in the price of new equipment and the relative stock returns

[7] A higher value of λ increases the volatility of investment-goods prices and lowers the volatility of investment growth, the volatility of the risk-free rate and the IMC portfolio.

of the investment- and consumption-goods producing sectors can be used as an empirical proxy for innovation in ω, which are driven by the investment shock. Furthermore, since the risk premium b_Z is negative, the model suggests that investment firms will have lower returns on average than consumption firms.

Finally, the value of growth opportunities to assets in place in the consumption sector increases as a function of ω, implying that $\zeta_G(\omega) > \zeta_V(\omega)$ in equations (36)-(37). Thus, the model generates intra-industry differences in risk premia between value and growth firms based on their differential exposure to the investment shock. Since the investment shock carries a negative price of risk b_Z, the model generates a positive average return differential between value and growth firms.

4.3 Model implications

I report the model-implied moments of consumption, investment and output growth, labor supply and the relative price of investment goods versus their empirical counterparts in the top and bottom panels of Table 2, respectively. The first three columns of the table report means, standard deviations and autocorrelation coefficients for each variable, while the last four columns report correlations among these variables. For the simulated data, the table shows the median values across simulations, along with the 5% and 95% percentile values in brackets. For the empirical moments, the table reports the point estimates, and the 90% confidence intervals in brackets.

For most of the moments of interest, the range of empirical estimates falls inside the 90% confidence intervals generated by the model. The volatility of both consumption and output growth is in line with their empirical counterparts, but the volatility of investment growth in the model is lower than the data (3.8% vs 6.2%). The model matches the amount of persistence in the data, as measured by the serial correlation of consumption, investment and output growth and investment goods prices. The model approximately matches the volatility (3.3% vs 3.0%) of the investment goods price series, though it generates a somewhat higher average growth rate for the price of investment goods (-2.8% vs -3.9%).

The model generates the right pattern of comovement between consumption, investment and output growth, with correlations ranging from 42% to 94%. The model also matches the correlation between the price of new equipment and consumption in the data (56% vs 44%), since consumption temporarily falls in response to a positive investment shock. However, the model fails to produce the right pattern of comovement between labor hours and consumption. The main reason is the specification of King et al. (1988) preferences over consumption and leisure: the strong income effect implied by these preferences results in labor supply being unresponsive to the consumption productivity shock.

Finally, like most general equilibrium models, the model has difficulty simultaneously matching the level of average consumption growth and the level of the risk free rate with a low value for the EIS. Thus, it matches the level of the risk-free rate but generates a somewhat lower average growth rate for consumption, output and investment than the data.

The model's ability to generate a realistic level of comovement between investment and consumption growth (42% vs 39% in the data) deserves further explanation, given that investment shocks generate opposite responses in investment and consumption. The positive correlation arises for two reasons. First, a positive productivity shock in the consumption sector A generates a positive response not only in consumption, but also in investment expenditures $I_t = p_t^I Y_{I,t}$, because the price of investment goods p^I is increasing in A. Second, the low value of the EIS implies that consumption does not fall very much in response to a positive investment shock. Given a low value for the EIS, the effect on consumption of the A shock dominates, generating a positive correlation between investment and consumption.

Table 3 compares the empirical moments of asset returns (first column) to the simulated moments from the benchmark calibration (second column). I consider the first two moments of the market portfolio, the risk-free rate, and the return spread between the investment- and consumption-goods sector. The model succeeds in generating a low and smooth risk free rate, as well as realistic risk premia and volatile stock returns. In the model, the equity premium equals 2.2%, compared to 4.9% in the data, while the average return spread between the investment- and consumption-goods producers equals -0.7%, compared to -1.4% in the data. Furthermore, the return differential between a pure value and pure growth firm equals 1.9%, which provides an upper bound on the value premium implied by the model. In the data, the value risk premium is approximately 6%, which is comparable in magnitude to the equity premium.

I decompose the risk premia generated by the model into the *amount* of risk and the *price* of risk, and compare them to the data. The model succeeds in capturing the amount of risk in the market portfolio, as its volatility is close to the data (21% vs 18%). However, the model undershoots in terms of the price of risk, as the Sharpe ratio of the market portfolio equals 11% compared to 27% in the data. By contrast, the model generates the right price of risk to justify the average return spread between investment- and consumption producers (-11% vs -13% in the data), but not the right amount of risk (volatility of 5.8% vs 11% in the data). This suggests that in order to match the moments of the return differential between investment- and consumption-firms, the model needs investment shocks that are twice as volatile.

The model is able to generate an equity premium of 2.2% with a risk aversion close to 1 and a volatility of consumption growth less than 2%. This result might be surprising in view of the large literature on the equity premium puzzle, but can be understood from examining equation (33). The equity premium in (33) can be decomposed into the risk premium for bearing risk due to the consumption- and investment-shocks. The compensation due to the consumption productivity shock is equal to $\gamma\sigma_A^2 = 0.04\%$, so it has a negligible effect in this calibration. Instead, most of the equity premium arises as compensation for the investment shock. In the model, a positive investment shock leads to an increase in the risk-free rate and consequently a contemporaneous fall in the market portfolio ($\zeta_M(\omega) < 0$). The investment shock contributes to the equity premium because the value of the market portfolio declines when real investment opportunities improve. This is consistent with the findings of Hobijn and Jovanovic (2001), who argue that the arrival of information technology caused a decline in the stock market in the 1970s.

4.4 The role of two investment shocks

The model specification in section 3 is not very common in that it features two investment shocks, Z_I and Z_m. By contrast, with the exception of Justiniano et al. (2011) most models feature a single investment shock, modeled either as a productivity shock to the investment sector (like Z_I) or as a shock to the relative price of equipment. In this section, I show that the model without the shock to the marginal efficiency of investment Z_m has difficulty simultaneously matching the volatility of the relative price of new equipment and the moments of asset returns and investment growth. Specifically, I solve the model for different values of $\sigma_{Z,I}$ while setting $\sigma_{Z,m} = 0$ and report results for these exercise in the third through fifth columns of Table 3.

Forcing the model to match the volatility of investment goods prices at $\sigma_{Z,I} = 4.5\%$ implies that the model does poorly at matching the first and second moments of asset returns. The equity premium implied by the model is 0.3%, while the average return differential between investment and consumption firms is -0.1%. The model generates an insufficient amount of risk in asset returns, as well as a low price of risk. In addition, the volatility of investment growth is too low compared to the data (2.0% vs 6.2%). Increasing the volatility of the investment shock to $\sigma_Z = 10\%$ modestly improves the model's ability to match asset pricing moments, at the cost of a substantially more volatile price of new equipment. In fact, a volatility of investment shocks of $\sigma_Z = 20\%$ is necessary for the model to generate realistic moments of asset returns. In this case, the equity premium is now close to the data at 4.4% and the average return differential between investment and consumption firms equals -1.1%. However, the volatility of investment growth is still lower than in the data

(5.0% vs 6.2%), although the difference is now smaller. By contrast, the price of equipment is now excessively volatile at 15%.

Increasing the volatility of the investment shock from 4.5% to 20% has a minimal impact on the volatility of consumption growth. The low elasticity of intertemporal substitution implies that the contemporaneous response of consumption growth to the investment shock is small. Since the investment shock accounts for a small fraction of the variance of short-term consumption growth, the correlation between investment and consumption growth is still positive at 30% even when the volatility of investment shocks is quite high.

In summary, the version of the model with only one investment shock Z_I has difficulty simultaneously matching the volatility of two price series: the relative price of investment goods (the price of uninstalled capital) and asset prices (the price of installed capital). We can see this tension by multiplying both sides of the first order condition (20) by the capital stock in the consumption sector K_C. Absent the shock to the marginal efficiency of investment Z_m, the right hand side corresponds to the stock market value of consumption firms. The model implies that the relative price of equipment p^I is about as volatile as asset prices.[8] However, the volatility of these two variables differs by an order of magnitude in the data. In order for the model to generate realistic asset prices, it needs to generate either a volatile price of new equipment, or introduce an additional wedge between the value of installed and uninstalled capital.

There are two reasons why forcing the model to match the volatility of investment goods prices may understate the importance of investment shocks. First, as Justiniano et al. (2011) point out, the presence of nominal rigidities in the production of investment goods may make the price of new equipment smoother than the underlying investment shock Z. Second, investment shocks can manifest as an improvement in the quality, rather than an increase in quantity of capital goods produced. If the price of new equipment is not adjusted for innovations in quality, this may understate the volatility of investment shocks.[9] For instance, the volatility of the relative price of computers, the only investment-goods price

[8]Choosing steeper adjustment costs (increasing λ) does not help make $c'(i_C)$ more volatile. When adjustment costs are steep, firms respond by choosing endogenously smooth investment i_C.

[9]Gordon (1990) constructs a quality adjusted price series for new equipment using hedonic regressions for the 1947-1983 period. Cummins and Violante (2002) and Israelsen (2010) regress Gordon's estimate (p^*) on the NIPA series (p_t) and GDP growth (Δy) for 22 categories of investment goods in the 1947-1983 period:

$$\ln p_t^* = c + \beta_1 t + \beta_2 \ln p_t + \beta_3 \ln p_{t-1} + \beta_4 \Delta y_t + e_t.$$

Using the estimated coefficients, they extrapolate the quality adjusted series until 2002, while Israelsen (2010) extends the series to 2008. This extrapolation may do a good job adjusting the mean growth rate, but it tends to understate the volatility of the quality adjusted price, if quality improvements do not occur at a constant rate. Furthermore, the presence of the lagged p_{t-1} term will induce additional smoothing.

that is quality-adjusted by the BEA using hedonic methods, is 8.7%, which is substantially higher than the volatility of the price series constructed by Gordon (1990), Cummins and Violante (2002) and Israelsen (2010).

The shock to the rate of transformation of investment goods into installed capital Z_m helps generate a smooth equipment price and volatile asset returns. By driving a wedge between the value of installed and uninstalled capital, it increases the volatility of asset returns but leaves the volatility of the price of new equipment relatively unaffected.

4.5 Extensions

The model presented thus far has two restrictions. First, the capital stock in the investment sector is fixed. Second, the model produces a counterfactual negative correlation between labor supply and consumption growth. I extend the model in turn to address each of these restrictions. I briefly describe the results here and reserve the details for the supplemental appendix.

Compared to the model in Section 3, the model with endogenous capital accumulation in the investment sector produces similar quantity dynamics, a higher equity premium and volatility of the risk-free rate and a smaller difference in risk premia between investment- and consumption-producers. Furthermore, since investment firms can purchase capital, there is a positive value of growth opportunities in the investment sector. Thus, in this extension, there is a difference in risk premia between value and growth firms in both the investment- and the consumption-producing sector.

The second extension considers preferences over consumption and leisure as in Jaimovich and Rebelo (2009). These preferences eliminate the strong short-run income effect in King et al. (1988), yet are still consistent with balanced growth. The extended model can now replicate the observed correlation between consumption and hours worked, while at the same time doing a slightly better job matching asset prices. This version of the model has an additional state variable affecting short-run fluctuations, which is a function of the consumption productivity shock A. Thus, in this case the consumption productivity shock A affects labor supply and the allocation of labor in the two sectors, generating a positive correlation between consumption and hours worked.

5 Empirical evidence

Here, I test the model's implications for asset prices in the data, using empirical measures of investment shocks. Specifically, I explore whether differences in securities' exposures to investment shocks leads to differences in risk premia. In addition, I explore whether differential exposure to investment shocks can rationalize the observed differences in average returns between value and growth firms.

5.1 Measuring investment shocks in the data

As discussed in Section 3.3, the model suggests two empirical proxies for investment shocks. The first measure is based on the relative price of new equipment, as in Greenwood, Hercowitz and Krusell [1997; 2000].[10] I construct this measure using the innovations in the relative price of investment goods Δz^I:

$$\ln p_t^I = a_0\, t + a_1\, t \cdot 1_{t \geq 1982} + \rho\, \ln p_{t-1}^I - \Delta z_t^I. \tag{38}$$

Thus, a positive investment shock ($\Delta z^I > 0$) is associated with a fall in the relative price of new equipment p^I. I allow the time trend to vary before and after 1982 because the price of new equipment experiences an abrupt increase in its average rate of decline in 1982 [Fisher, 2006]. Results are quantitatively similar across different specifications for Δz^I, for example using $\Delta z_t^I = -\Delta \ln p_t^I$ or omitting the differential time trend in equation (38).

In the model, the price of new equipment is a noisy proxy for investment shocks, since p^I also depends on the productivity shock in the consumption sector A. Thus, I also construct a second measure of investment shocks: the stock return spread between investment- and consumption good-producers. Equations (29) and (30) imply that the cross-section of asset returns responds differentially to an investment shock. Thus, asset returns can also be used to infer realizations of Z in the data. I construct the equivalent of the investment and consumption industry portfolios in the data following Gomes et al. (2009).

I display the composition details of the two industry portfolios in Table 4. The sector producing consumption goods is larger than the sector producing investment goods, both in number of firms and in terms of market capitalization. However, the consumption and investment portfolio have fairly similar ratios of book to market equity, debt to assets and cashflows to assets. The relative size of the investment relative to the consumption sector, in terms of total market capitalization, is around 23%, although this ratio varies substantially over time. Figure 2 shows that the ratio of market capitalization of investment to consumption producers displays significant comovement with the ratio of investment to consumption

[10] Greenwood et al [1997; 2000] consider a one-sector version of the model, where the investment shock enters into the capital accumulation equation. In their model, it corresponds exactly to the reciprocal of the relative price of investment goods. In my two sector specification, the relative price of investment goods p^I is endogenous and given by equation (20).

expenditures in the economy. The correlation between the logarithm of the two series is 58% in levels and 31% in first differences.

Because the return spread between investment- and consumption-good producers (IMC) is a new measure of investment shocks, it is important to first establish the validity of this measure. Specifically, I estimate the dynamic response of output, consumption, investment, labor hours and the relative price of new equipment to returns on the IMC portfolio, and compare the estimated responses to those implied by the model. I estimate

$$\frac{1}{1+k}\left(x_{t+k}-x_{t-1}\right)=\alpha_0+\beta_k R_t^{imc}+\gamma_k \Gamma_t+e_{tk}, \qquad k=0\dots K \tag{39}$$

where x_t denotes the log value of the predicted variable, and R_t^{imc} denotes the return spread between investment- and consumption-good producers. Γ_t is a vector of controls, which includes one lag of Δx_t. Empirically, the IMC return spread is positively correlated with the market portfolio, which is known to predict business cycle variables. I control for this by including returns to the market portfolio in the vector of controls Γ_t when estimating equation (39) in the data. I normalize all variables to zero mean and unit standard deviation. I estimate equation 39 using quarterly data in the 1951:2008 period.[11] I examine horizons up to $K = 8$ quarters ahead. I adjust the standard errors for heteroscedasticity and serial correlation using the Newey-West procedure, with the maximum lag length equal to the number of overlapping quarters plus two. In simulated data, I calculate the median estimated coefficient β_k and the 5% and 95% percentiles across 10,000 simulations, each with a length of 50 years.

The empirical results support using the return spread between investment and consumption producers as a measure of the investment shock. The response of macroeconomic variables to returns of the IMC portfolio is similar in the data and the model, as we can see in the top and bottom panels of Figure 3, respectively. Both in the data and the model, positive returns to the IMC portfolio lead to a decline in consumption in the short run, and an increase in labor supply. In addition, investment sharply increases in response to the IMC portfolio, whereas the the relative price of new equipment falls. By contrast, the estimated long-run response of consumption growth is not statistically significant from zero in the model or the data. This is hardly surprising, given that the sample contains very few long-horizon observations that are independent.

[11] Since the quality adjusted price series of new equipment of Israelsen (2010) and Gordon (1990) is only available at annual frequencies, I use the NIPA deflator for equipment and software divided by the consumption deflator for nondurables as a measure of the price of investment goods.

5.2 Asset prices

Methodology

The model's main predictions for asset prices are that: i) the investment shock causes investment firms to appreciate in value relative to consumption firms (i.e., $\zeta_I(\omega) > \zeta_C(\omega)$); ii) the investment shock causes the value of growth opportunities to appreciate relative to the value of assets in place (i.e. $\zeta_G(\omega) > \zeta_V(\omega)$); and iii) the investment shock carries a negative price of risk (i.e. $b_Z(\omega) < 0$). I use the model's first prediction to construct my second measure of investment shocks, and test the other two independently.

Testing the second prediction of the model requires measuring the relative response of the value of growth opportunities and the value of existing assets to a positive investment shock. However, the value of assets in place and growth opportunities are not observable in the data. Thus, I examine instead whether a positive investment shock leads to an appreciation in the price of firms which derive most of their value from growth opportunities relative to the price of firms whose value is mostly attributable to existing assets. I use the firm's ratio of book-to-market equity to measure the relative contribution of growth opportunities and assets in place in firm value, since firms with lower book-to-market ratios likely derive most of their value from growth opportunities.[12] Because firms in the investment sector in the benchmark model have no growth opportunities, I test the empirical predictions on firms in the consumption sector only.[13]

I test the model's third prediction by estimating a linear approximation of the stochastic discount factor (26) around the stationary mean $\overline{\omega}$ of the state variable ω :

$$m = b_0 - b_A \Delta\tilde{a} - b_Z \Delta\tilde{z}, \tag{40}$$

where $\Delta\tilde{a}$ and $\Delta\tilde{z}$ represent standardized innovations to the consumption and investment shock respectively. The approximation is equivalent to assuming a constant risk premium for the investment shock $b_Z(\omega) \approx b_Z(\overline{\omega})$. This approximation is fairly accurate because the sensitivity of $b_Z(\omega)$ to ω is low. I use proxies for the investment ($\Delta\tilde{z}$) and the consumption productivity ($\Delta\tilde{a}$) shocks and estimate the parameters b_A and b_Z in the data. For the investment shock Z, I use the two proxies constructed in section 5.1: innovations in the

[12] Consistent with this view, low book-to-market firms have substantially higher investment rates than high book-to-market firms in the data. Alternatively, sorting firms on Tobin's Q produces very similar results. I sort stocks on book to market rather than Tobin's Q because book-to-market differences lead to portfolios that are mispriced by the CAPM and the consumption CAPM (the value premium puzzle).

[13] In section (4.5) I extend the model to allow endogenous capital accumulation in the investment sector. Consistent with the predictions of this model, replicating the empirical analysis in the investment sector leads to similar conclusions to those obtained in this section (see Supplemental Appendix for details).

relative price of new equipment Δz^I and returns on the IMC portfolio. I measure innovations to the consumption productivity shock A using returns on the CRSP value-weighted portfolio and non-durable consumption growth plus services from NIPA. I also estimate specifications restricting $b_Z = 0$, which given my proxies for A, correspond to the CAPM and Consumption CAPM respectively.

I estimate the parameters of the stochastic discount factor (40) using the generalized method of moments (GMM). I use the moment restrictions on the excess rate of return of any asset that is imposed by no arbitrage:

$$E[mR_i^e] = 0. \tag{41}$$

In my estimation, I use portfolio returns in excess of the risk-free rate R_i^e, so the mean of the stochastic discount factor m is not identified from the moment restrictions (41). I thus choose the normalization $E(m) = 1$. Using this normalization, the moment conditions (41) become

$$E[R_i] - r_f = -cov(m, R_i^e), \tag{42}$$

which is the empirical equivalent of equation (25), but with the conditional moments replaced by their unconditional counterparts. I evaluate the model's ability to price these assets correctly based on the residuals of the Euler equation (42). I compute the sum of squared pricing errors (SSQE) and the J-test of the over-identifying restrictions of the model, namely that all the pricing errors are zero. Finally, I adjust the standard errors of the GMM estimator using the Newey-West estimator, allowing for a maximum lag length of 3 years.

Using the entire cross-section of stock returns to estimate the stochastic discount factor (40) can be problematic because covariances are measured with error. To reduce measurement error, most studies focus on portfolios of stocks sorted on economically meaningful characteristics. Estimating the price of risk b_Z accurately requires assets with substantial dispersion in their exposure to the investment shock. Thus, I consider the following two sets of assets: First, I create portfolios of firms sorted on their past sensitivity to the investment shock. Using low frequency data, it is difficult to accurately estimate time-varying firm-level sensitivities to the investment shock. Thus, I focus on the return spread between investment and consumption firms as my measure of investment shocks, and estimate the firm-level sensitivities using weekly data on stock returns. As long as these sensitivities are fairly persistent, this procedure will generate ex-post differential sensitivity to the investment shock across these portfolios. In a second exercise, I explore prediction (ii) and use the cross-section of firms sorted by their book-to-market ratio. For conciseness, I describe the details of the construction of these portfolios in Appendix B.

Results

The top panel of Table 5 shows the average excess returns and risk characteristics for the 10 portfolios of firms sorted on their sensitivity with the investment shock. First, these portfolios display a declining pattern of average excess returns, ranging from 6.5% to 4.4% per year. Second, these portfolios display an increasing pattern of covariances with both empirical measures of the investment shock. The difference in investment shock sensitivities between the two extreme portfolios has a t-statistic of 2.1 and 10.7 depending on whether the investment shock is measured as innovations to the equipment price or as the return spread between investment and consumption firms. Third, there is an increasing pattern of risk sensitivities with respect to the market portfolio or aggregate consumption. Since standard models like the CAPM and the CCAPM assign a positive risk-price to the market portfolio and consumption growth respectively, they will fail to price these portfolios.[14]

The bottom panel of Table 5 repeats the above exercise for the 10 portfolios sorted on book-to-market. These portfolios have an increasing pattern of average excess returns, ranging from 3.9% to 10% a year, while essentially having the same or declining market betas. This pattern is the well-studied value premium puzzle. Consistent with the second prediction of the model, portfolios of firms sorted on book-to-market display a declining pattern of exposures with both measures of the investment shock. Sensitivities to the IMC portfolio decline from 0.4 to 0.2 across the extreme portfolios, while sensitivities to Δz^I fall from -2.4 to -3.4. The t-statistics on the difference across extreme portfolios are -1.2 and -1.8 respectively. These numbers somewhat understate the decline in risk exposures, mainly due to the behavior of the extreme value portfolio. If instead we focus on the return spread between portfolios 1 and 9, the difference in exposures to the IMC spread and Δz^I are equal to -0.4 and -1.3, with t-statistics of -2.1 and -1.7 respectively.

I estimate the risk premium of the investment shock to be negative and statistically significant. Table 6 presents the results of estimating the parameters of the stochastic discount factor (40) via GMM. The top panel presents results using the 10 portfolios of firms sorted on investment-shock sensitivities. The estimates of the risk premium $\hat{b}_Z$ range from -0.66 to -0.80 when using the innovations to the equipment price Δz^I, and -0.29 to -0.47 when using the return spread between investment and consumption firms (IMC). In terms of asset pricing errors, including either measure of investment shocks to the standard model reduces the sum of squared errors to $0.02-0.17$ relative to $0.37-0.78$ when using the CAPM or the

[14]For instance, the return spread between the two extreme portfolios has a CAPM alpha of -5.6% with a t-statistic of -2.2. This CAPM alpha implies that there exists a linear combination of the market portfolio and the high- and low-IMC-beta portfolio that has an average return of 5.6% in excess of the risk-free rate and zero exposure to the market portfolio.

consumption CAPM.

The results are similar when using the cross-section of 10 book-to-market portfolios. The estimates of the risk premium on the investment shock $\hat{b}_Z$ range from -1.43 to -1.52 when using the relative price of new equipment to -0.14 to -0.77 when using the investment-consumption return spread. The estimates are, with one exception, statistically different from zero. Furthermore, both measures of investment shocks help reduce the pricing errors of the CAPM and CCAPM from $0.27 - 0.33$ to $0.12 - 0.26$, depending on the specification. Thus, the data support the view that value firms earn higher risk premia than growth firms because their relative stock price declines following a positive investment shock.

In summary, the empirical estimates of the risk premium of the investment shock $\hat{b}_Z$ vary from -0.14 to -1.52. I compare these estimates to the price of risk $b_Z(\omega)$ implied by the model (27). Using the stationary distribution of ω, the unconditional risk premium associated with the investment shock $E[b_Z(\omega)]$ equals -0.12. Part of the reason for this difference could be due to estimation noise. If the investment shock is measured with error, the estimated covariance between stock returns and the investment shock will be biased towards zero, leading to inflation in the estimated risk premium b_Z.

The results of this section can also be seen graphically. Figure 4 plots the average returns of these portfolios versus their covariance with the investment shock. There is a strong negative relationship between average returns on these portfolios and their sensitivities to either measure of the investment shock. This is consistent with prediction (iii): investors attach a higher value to securities that pay off when investment opportunities are good, and are thus willing to accept lower risk premia for holding them.

As an additional robustness test, I also estimate the parameters of the stochastic discount factor (40) using a set of assets that are more standard in the finance literature: the cross-section of 25 Size and Book to Market Portfolios of Fama and French (1993) and the 30 Industry portfolios of Fama and French (1997). This exercise also illustrates the extent to which the two empirical measures of investment shocks can explain differences in risk premia among these assets. I present these estimation results in Table 7. The top panel shows results for the 25 portfolios sorted on market capitalization and book-to-market equity. The estimated risk premium on the investment shock is negative and statistically different from zero, and ranges from -0.32 to -1.56 depending on the specification. Including either measure of the investment shock helps price these portfolios, as the sum of squared errors drops by a factor of three. The bottom panel shows estimation results for the cross-section of 30 Industry portfolios. In this case, the estimated premium on the investment shock is negative and statistically different from zero in two out of the four cases, and ranges from -0.13 to -0.52 depending on the specification. Thus, these results provide additional validation to the view that a positive investment shock leads to high marginal utility of wealth states.

6 Conclusion

This paper explores the implications of a real business cycle model with investment shocks for asset prices. Investment shocks capture the idea that some technological innovations affect output only to the extent that they are implemented through the formation of new capital stock. A positive investment shock causes a fall in the resource cost of producing new capital, leading to an increase in real investment opportunities.

There are two main new points in this paper. First, investment shocks affect the marginal utility of the representative household, since a positive investment shock leads to a reallocation of resources from the consumption to the investment sector. Depending on the preference of households over smoothing consumption across time (given by the elasticity of intertemporal substitution) and smoothing across states (captured by the coefficient of risk aversion), states of the world when real investment opportunities are good can be either high or low marginal utility states. Empirical evidence supports the former, since securities whose returns are positively correlated with the investment shock earn a lower risk premium.

Second, investment shocks have a heterogenous effect on asset returns, thus leading to cross-sectional differences in asset risk premia, both between and within industries. Specifically, a positive investment shock benefits producers of capital goods relative to producers of consumption goods, and within each sector, benefits firms with opportunities to invest relative to those that lack growth opportunities. Thus, my model implies that growth firms can earn lower risk premia than value firms because they act as hedges for shocks to real investment opportunities.

My results offer new insights about the link between the cross-section of asset returns and macroeconomic quantities. If macroeconomic disturbances do not affect firms symmetrically, then the cross-section of asset returns can help with the identification of fundamental shocks. Financial data offer certain advantages relative to real economic quantities, in that they are forward looking and are available at high frequencies. Furthermore, the cross-section of asset risk premia is informative about the properties of these macroeconomic shocks and their effect on the marginal utility of wealth of the representative household.

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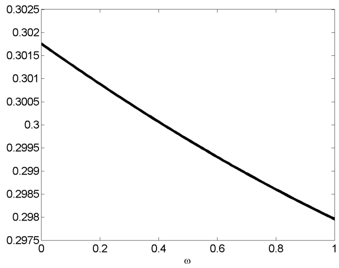
7 Tables and figures

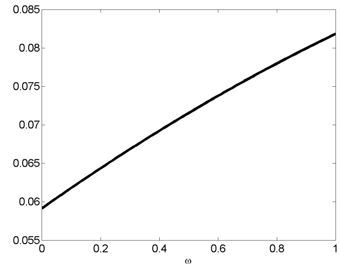
Table 1: Parameters used for benchmark calibration

Parameter	Symbol	Value
Preferences		
Discount rate	ρ	0.001
Elasticity of intertemporal Substitution	θ	0.3
Relative risk aversion	γ	1.1
Share of leisure in utility	ψ	3
Technology		
Growth rate of productivity shock in consumption sector	μ_A	0.1%
Volatility of productivity shock in consumption sector	σ_A	2.0%
Growth rate of productivity shock in investment sector	μ_Z	4.0%
Volatility of productivity shock in investment sector	$\sigma_{Z,I}$	3.5%
Volatility of shock to the marginal efficiency of investment	$\sigma_{Z,m}$	13.5%
Production		
Capital elasticity in C-sector	β_C	0.3
Capital elasticity in I-sector	β_I	0.3
Adjustment cost parameter	λ	1.15
Depreciation rate of capital	δ	8.5%
Other		
Financial Leverage	-	5/3

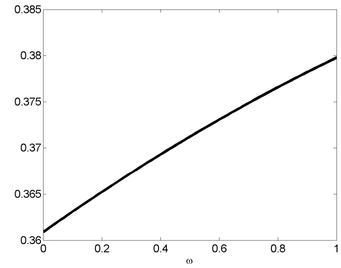
Figure 1: Model Solution



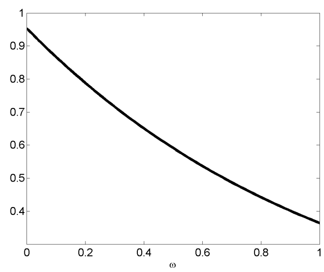
i) Consumption (C)



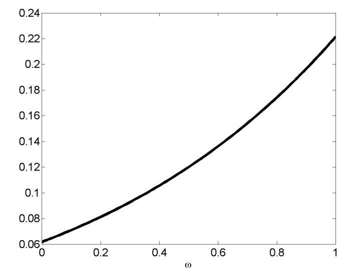
ii) Investment ($p^I\, Y_I$)



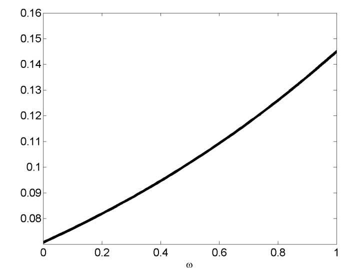
iii) Output ($C + p^I\, Y_I$)



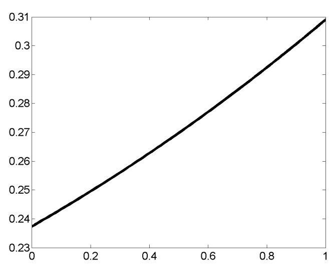
iv) Price of investment goods relative to MEI shock (p^I/Z_m)



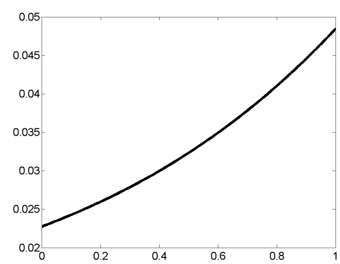
v) Investment rate (i_C)



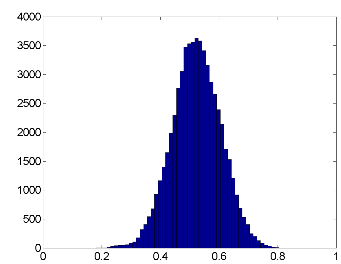
vi) Marginal utility (h_C)



vii) Value of investment firms relative to consumption firms (S_I/S_C)



viii) Value of growth relative to assets in place in consumption sector (S_{CG}/S_{CV})



ix) Stationary distribution of ω

Table 1 plots the numerical solution of the model. I evaluate the above aggregate quantities and prices at $K_C = 1$ and $A = 1$, and plot them as a function of ω. I obtain the distribution of ω by simulating one long path from the model of 10,000 years. I drop the first half to remove the dependence on initial values.

Table 2: Model versus data: macroeconomic quantities

	Mean	Volatility	AR(1)	Correlation $\dot{c}$	Correlation $\dot{i}$	Correlation $\dot{l}$	Correlation $\dot{y}$
A: Data							
$\dot{c}$	2.51	1.95	0.40				
	[2.09, 2.93]	[1.65, 2.24]	[0.21, 0.58]				
$\dot{i}$	2.60	6.22	0.17	0.39			
	[1.26, 3.94]	[5.26, 7.18]	[-0.04, 0.38]	[0.21, 0.58]			
$\dot{l}$	-0.08	2.52	0.16	0.41	0.83		
	[-0.62, 0.47]	[2.13, 2.13]	[-0.06, 0.37]	[0.23, 0.59]	[0.76, 0.90]		
$\dot{y}$	2.35	3.24	0.10	0.84	0.67	0.64	
	[1.66, 3.05]	[2.74, 3.74]	[-0.12, 0.32]	[0.78, 0.90]	[0.55, 0.79]	[0.51, 0.76]	
$\dot{p}^I$	-3.78	3.01	0.18	0.44	-0.06	-0.26	0.24
	[-4.42, -3.13]	[2.55, 3.47]	[-0.03, 0.39]	[0.26, 0.61]	[-0.28, 0.16]	[-0.46, -0.05]	[0.03, 0.44]
B: Model							
$\dot{c}$	1.11	1.87	0.39				
	[0.12, 2.07]	[1.58, 2.31]	[0.15, 0.60]				
$\dot{i}$	1.11	3.82	0.24	0.42			
	[-0.12, 2.08]	[3.10, 4.66]	[0.00, 0.41]	[0.22, 0.62]			
$\dot{l}$	0.00	0.80	0.53	-0.08	0.87		
	[-0.07, 0.07]	[0.65, 0.94]	[0.36, 0.68]	[-0.29, 0.16]	[0.80, 0.92]		
$\dot{y}$	1.11	1.94	0.38	0.94	0.71	0.26	
	[0.10, 2.07]	[1.61, 2.32]	[0.12, 0.59]	[0.90, 0.96]	[0.58, 0.82]	[0.06, 0.47]	
$\dot{p}^I$	-2.79	3.26	0.25	0.56	0.21	-0.10	0.53
	[-3.85, -1.66]	[2.71, 3.93]	[-0.05, 0.50]	[0.34, 0.70]	[-0.03, 0.36]	[-0.30, 0.11]	[0.30, 0.65]

Table 2 compares moments of the data to simulated moments from the model. I consider log output growth $\dot{y}$; consumption growth $\dot{c}$; investment growth $\dot{i}$; labor supply growth $\dot{l}$; growth of investment goods prices $\dot{p}^I$. The top panel shows moments in actual data, estimated using annual data in the 1951:2008 period. The bottom panel shows moments from simulated data. I simulate the model at a monthly frequency and then time-aggregate the data to form annual observations. I report median moments along with the 5% and 95% percentiles across 10,000 simulations, each with a length of 50 years.

Table 3: MODEL VERSUS DATA: ASSET PRICES

Moments	Data	Model Bench.	Model Alt. 1	Model Alt. 2	Model Alt. 3
Risk premium of market portfolio	4.89	2.18	0.30	1.12	4.42
Volatility of market portfolio	17.92	20.55	7.32	14.87	29.22
Risk premium between i- and c-firms	-1.41	-0.68	-0.10	-0.32	-1.13
Return spread volatility between i- and c-firms	10.96	5.80	2.00	4.30	7.60
Mean of risk-free rate	2.90	2.83	4.19	3.60	1.22
Volatility of risk-free rate	3.00	3.61	1.23	2.67	4.72
Risk premium between pure value and growth firms	6.15	1.87	0.18	0.98	3.88
Return spread volatility between pure value and growth firms	19.12	18.27	5.78	13.70	26.65
Volatility of consumption growth	1.95	1.87	1.64	1.75	2.00
Volatility of investment growth	6.22	3.82	1.97	2.97	5.04
Volatility of equipment price	3.05	3.26	3.79	7.82	15.48
Correlation between consumption and investment growth	0.39	0.42	0.82	0.54	0.31
Correlation between output and investment growth	0.67	0.71	0.89	0.75	0.69
Correlation between consumption and output growth	0.84	0.94	0.99	0.96	0.90
Volatility of investment productivity shock σ_{ZI}		3.5	4.5	10	20
Volatility of shock to marginal efficiency of investment σ_{Zm}		13.5	-	-	-

Table 3 compares moments of the data (column one) to simulated moments from the model (columns two to five). Column two shows moments from simulated data generated from the benchmark model with both investment shocks. Columns three to five show moments from simulated data generated from a model with only the productivity shock in the investment sector ($\sigma_{Z,m} = 0$) for different values of $\sigma_{Z,I}$. All numbers except correlations are in percentage terms. I simulate the model at a monthly frequency and then time-aggregate the data to form annual observations. I report median values along with 5% and 95% percentiles across 10,000 simulations, each with a length of 50 years. The moments for stock returns are computed over the 1962:2008 period. The mean of the real risk-free rate are from the long sample of Campbell and Cochrane (1999). The volatility of the real interest rate is from Chan and Kogan (2002). There is no equivalent to a pure value and growth firm in the data, so I report the mean and volatility of the return spread between two portfolios of firms with the top and bottom decile of book-to-market ratios. See notes to Table 2 for definitions of macroeconomic variables.

Table 4: IMC PORTFOLIO: COMPOSITION

	Consumption Portfolio				Investment portfolio			
	mean-vw	median	10%	90%	mean-vw	median	10%	90%
ME ($b)		2.35	0.21	25.93		1.47	0.16	16.03
Book-to-Market Equity	0.62	0.72	0.26	1.80	0.53	0.62	0.20	1.66
Debt-to-Asset	0.23	0.23	0.01	0.51	0.17	0.16	0.01	0.41
Cashflows-to-Assets	0.10	0.07	-0.16	0.16	0.12	0.07	-0.06	0.46
Number of firms	3053				1015			

Table 4 reports composition details for the investment minus consumption portfolio (IMC) constructed using the NIPA tables. I report the market value of equity (ME) in 2005 dollars, the book-to-market ratio (COMPUSTAT item ceq over ME), financial leverage (COMPUSTAT items dltt + dlc over at), and cashflows to assets (COMPUSTAT item ib plus item dp over item at). I report time-series averages of the market-capitalization weighted average within each portfolio (mean-vw), the median and the 10% and 90% decile within each portfolio. Sample includes data from 1962-2008.

Figure 2: STOCK PRICES OF I AND C FIRMS VERSUS INVESTMENT-TO-CONSUMPTION RATIO

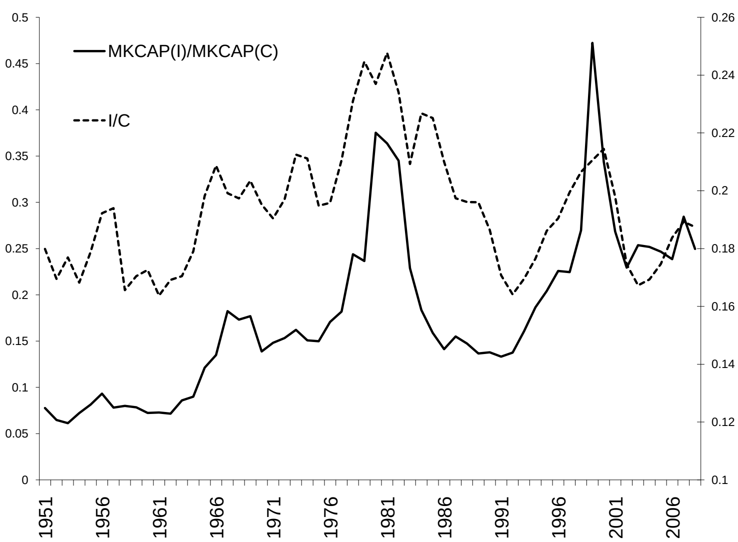
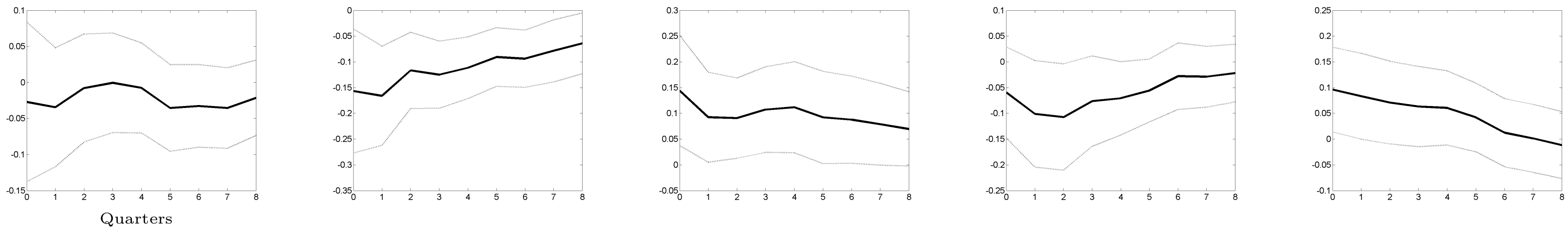


Figure 2shows the ratio of market values of investment over consumption industries (left axis) and the ratio of private fixed nonresidential investment over the sum of personal consumption expenditures in non-durables and services (right axis).

Figure 3: Response to IMC (Data vs Model)

Output (Y)	Consumption (C)	Investment (I)	Equipment Price (p^I)	Labor Hours (H)

Panel A: Data



Panel B: Model

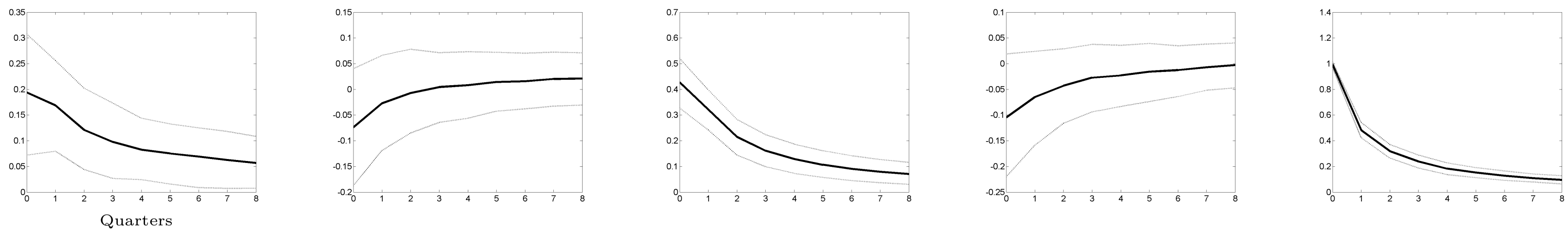


Figure 3 plots the response of macroeconomic variables to the return spread between investment- and consumption-good producers. I estimate Equation 39 in actual and simulated data. Panel A shows the regression results in actual data, where the estimation sample includes quarterly data in the 1951:2008 period. The definitions of macroeconomic variables are in Table 2, except that I measure p^I using the NIPA deflator for equipment and software. I plot the estimated β_k coefficients along with 90% confidence intervals based on HAC standard errors using the Newey-West kernel, with bandwidth equal to the length of the overlap plus two quarters. Panel B shows the regression results in simulated data. I simulate the model at a monthly frequency (dt=1/12) and then aggregate the data to form quarterly observations. I plot the median β_k coefficients along with the 5% and 95% percentiles across 10,000 simulations, each with a length of 50 years.

Table 5: EXCESS RETURNS AND COVARIANCES OF 10 PORTFOLIOS SORTED ON IMC BETA AND BOOK-TO-MARKET

	A: 10 portfolios sorted on IMC beta (Consumption firms only)										
IMC Beta	Low	2	3	4	5	6	7	8	9	High	High - Low
Mean excess return (μ)	6.47	6.11	6.61	6.62	6.14	5.77	5.37	5.36	5.27	4.36	-2.11
	(2.34)	(2.25)	(2.68)	(2.87)	(2.28)	(2.01)	(1.88)	(1.63)	(1.30)	(0.88)	(-0.46)
Volatility (σ)	18.52	18.19	16.55	15.44	18.07	19.27	19.16	22.01	27.24	33.20	30.70
Sharpe ratio (μ/σ)	34.92	33.59	39.98	42.84	33.99	29.94	28.01	24.37	19.35	13.13	-6.87
Risk exposure to A ($\dot{c}$)	3.51	2.25	2.46	1.57	3.03	3.16	4.15	3.33	4.86	6.99	3.48
	(2.20)	(1.11)	(1.75)	(0.94)	(1.90)	(1.74)	(2.19)	(1.77)	(1.74)	(2.00)	(0.99)
Risk exposure to A (R_{mkt})	0.80	0.89	0.80	0.75	0.96	1.03	1.01	1.12	1.34	1.48	0.68
	(6.60)	(11.73)	(11.76)	(9.07)	(15.22)	(20.40)	(34.78)	(20.48)	(12.21)	(7.48)	(2.23)
Risk exposure to Z (Δz^I)	-3.41	-2.80	-2.60	-2.37	-2.77	-3.18	-1.67	-2.47	-1.38	-0.51	2.90
	(-2.47)	(-2.38)	(-2.60)	(-2.67)	(-2.16)	(-2.40)	(-1.06)	(-1.53)	(-0.70)	(-0.20)	(2.14)
Risk exposure to Z (R_{imc})	-0.07	-0.03	-0.07	-0.11	0.23	0.25	0.60	0.61	1.25	1.77	1.84
	(-0.36)	(-0.21)	(-0.69)	(-0.95)	(1.89)	(1.60)	(5.68)	(2.80)	(9.92)	(10.27)	(10.67)
	B: 10 portfolios sorted on book-to-market (Consumption firms only)										
Book-to-market	Low	2	3	4	5	6	7	8	9	High	High - Low
Mean excess return (μ)	3.88	5.93	4.66	5.63	5.86	6.79	6.60	8.89	9.18	10.04	6.15
	(1.29)	(2.25)	(1.73)	(2.26)	(2.22)	(2.45)	(2.28)	(3.04)	(2.77)	(2.98)	(2.16)
Volatility (σ)	20.16	17.68	18.09	16.73	17.72	18.57	19.45	19.60	22.20	22.62	19.12
Sharpe Ratio (μ/σ)	19.27	33.54	25.75	33.68	33.08	36.57	33.92	45.37	41.33	44.37	32.17
Risk exposure to A (R_{mkt})	1.01	0.91	0.94	0.85	0.88	0.89	0.91	0.92	1.00	1.03	0.02
	(19.30)	(27.98)	(13.71)	(11.71)	(10.55)	(8.36)	(7.36)	(8.48)	(9.70)	(9.79)	(0.17)
Risk exposure to A ($\dot{c}$)	2.40	1.99	2.39	2.83	2.73	3.18	4.05	2.67	3.59	3.42	1.02
	(1.11)	(1.23)	(1.38)	(1.91)	(1.72)	(1.78)	(2.30)	(1.68)	(1.75)	(1.63)	(0.72)
Risk exposure to Z (Δz^I)	-2.36	-2.14	-2.41	-2.60	-2.47	-3.38	-3.05	-3.30	-3.62	-3.35	-0.99
	(-1.71)	(-1.71)	(-1.62)	(-2.18)	(-2.35)	(-2.14)	(-2.54)	(-2.47)	(-3.36)	(-2.47)	(-1.78)
Risk exposure to Z (R_{imc})	0.38	0.23	0.23	0.23	0.05	0.06	-0.00	-0.05	0.02	0.19	-0.19
	(2.51)	(1.79)	(1.43)	(1.48)	(0.33)	(0.30)	(-0.02)	(-0.34)	(0.12)	(1.19)	(-1.24)

Table 5 reports summary statistics for simple excess returns over the 30-day T-bill rate for two sets of portfolios created by using only firms in the consumption industry. I report mean excess returns over the risk-free rate (μ) and volatilities (σ). I also report risk exposures (univariate betas) with respect to measures of the consumption productivity shock A and the investment shock Z. I use two proxies for the consumption productivity shock A: returns on the market portfolio (MKT) and the growth rate of per capita non-durables plus services consumption (C). I use two proxies for Z: minus the innovation in the relative price of new equipment (Δz^I) using Equation 38, and returns on the portfolio of investment minus consumption producers IMC. Panel A shows results for 10 portfolios of firms sorted on IMC exposure (β^{IMC}). Panel B shows results for 10 portfolios of firms stocks on their book-to-market ratio. Sample includes annual data from 1963:2008. I report t-statistics in parentheses using Newey-West standard errors.

Table 6: Cross-Sectional Tests (I)

Panel A: 10 portfolios sorted on IMC-beta (Consumption Sector only)						
Factor	CAPM	CCAPM	IMC, MKT	IMC, C	p^I, MKT	p^I, C
$A:\ R_{mkt}$	0.29		0.41		0.14	
	[2.34]		[3.16]		[0.81]	
$A:\ \dot{c}$		0.83		1.44		0.32
		[2.17]		[3.09]		[0.78]
$Z:\ R_{imc}$			-0.29	-0.47		
			[-1.80]	[-2.46]		
$Z:\ \Delta z^I$					-0.66	-0.80
					[-1.75]	[-2.73]
$SSQE(\%)$	0.37	0.78	0.02	0.17	0.06	0.08
J-test	12.3	24.4	5.4	17.8	11.7	14.4
	[0.20]	[0.00]	[0.71]	[0.02]	[0.17]	[0.07]
Panel B: 10 portfolios sorted on book-to-market equity (Consumption Sector only)						
Factor	CAPM	CCAPM	IMC, MKT	IMC, C	p^I, MKT	p^I, C
$A:\ R_{mkt}$	0.40		0.48		-0.11	
	[3.18]		[3.40]		[-0.45]	
$A:\ \dot{c}$		1.41		1.46		-0.50
		[3.20]		[3.34]		[-1.47]
$Z:\ R_{imc}$			-0.77	-0.14		
			[-1.97]	[-0.41]		
$Z:\ \Delta z^I$					-1.43	-1.52
					[-2.03]	[-4.99]
$SSQE(\%)$	0.33	0.27	0.16	0.26	0.12	0.12
J-test	53.3	63.4	34.6	57.4	35.3	18.0
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.02]

Table 6 shows results of estimating the stochastic discount factor of the model, (40), via generalized method of moments. I report first stage estimates of b_A and b_Z using the identity weighting matrix. I also report the sum of squared pricing errors (SSQE) and the J test of over-identifying restrictions along with p-values in brackets. I report t-statistics in parentheses using Newey-West standard errors, allowing for 1 lag. I use two proxies for the consumption productivity shock A: returns on the market portfolio (MKT) and the growth rate of per capita non-durables plus services consumption (C). I use two proxies for Z: minus the innovation in the relative price of new equipment (Δz^I) using Equation 38, and returns on the portfolio of investment minus consumption producers IMC. I normalize both proxies for Z to have unit standard deviation. The top panel (A), shows estimates of the parameters using a cross-section of 10 portfolios sorted on their univariate beta with respect to the IMC portfolio (β^{IMC}). The bottom panel (B) shows estimates of the parameters using a cross-section of 10 portfolios sorted on their book-to-market ratio. Sample includes annual data from 1963:2008.

Figure 4: Historical average returns versus covariances with I-shock

A: 10 IMC-beta sorted portfolios (Consumption Sector only)

B: 10 book-to-market sorted portfolios (Consumption Sector only)

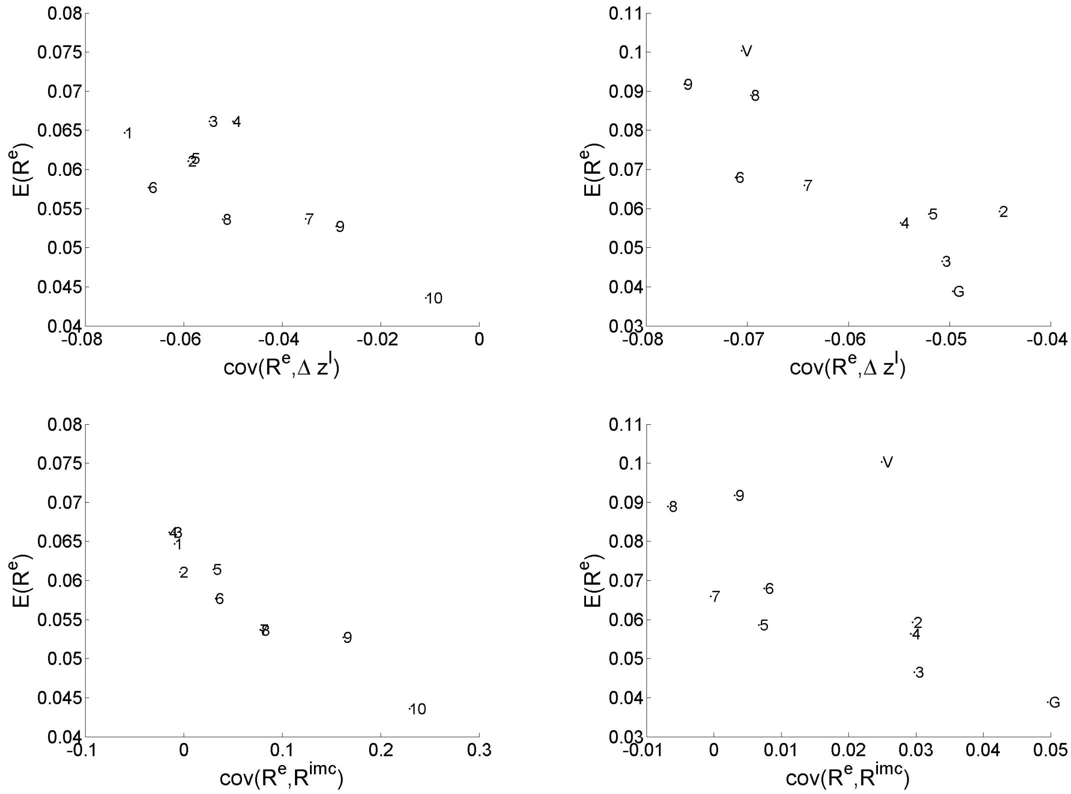


Figure 4 plots average returns in excess of the risk-free rate $E(R^e)$ versus covariances with the investment-specific shock (Z) for two sets of portfolios. I use two proxies for Z: minus the innovation in the relative price of new equipment (Δz^I) using Equation 38, and returns on the portfolio of investment minus consumption producers (R_{imc}). The left panel (A), uses 10 portfolios sorted on their univariate beta with respect to the IMC portfolio (β^{IMC}). The right panel (B) uses 10 portfolios sorted on their book-to-market ratio. Sample includes annual data from 1963:2008.

Table 7: CROSS-SECTIONAL TESTS (II)

Panel A: 25 portfolios sorted on market-equity and book-to-market						
Factor	CAPM	CCAPM	IMC, MKT	IMC, C	p^I, MKT	p^I, C
$A:\ R_{mkt}$	0.44		0.62		-0.14	
	[3.25]		[4.24]		[-0.95]	
$A:\ \dot{c}$		1.74		1.98		0.46
		[3.48]		[4.32]		[1.09]
$Z:\ R_{imc}$			-0.81	-0.32		
			[-4.51]	[-1.76]		
$Z:\ \Delta z^I$					-1.56	-0.91
					[-4.65]	[-2.16]
$SSQE(\%)$	3.69	3.05	1.12	2.49	1.93	1.90
J-test	113.4	130.8	83.8	101.1	110.0	115.9
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Panel B: 30 industry portfolios						
Factor	CAPM	CCAPM	IMC, MKT	IMC, C	p^I, MKT	p^I, C
$A:\ R_{mkt}$	0.36		0.41		0.31	
	[2.31]		[3.32]		[2.13]	
$A:\ \dot{c}$		1.05		1.54		0.54
		[3.03]		[3.49]		[2.01]
$Z:\ R_{imc}$			-0.20	-0.45		
			[-1.33]	[-2.48]		
$Z:\ \Delta z^I$					-0.13	-0.52
					[-0.60]	[-2.71]
$SSQE(\%)$	2.38	5.83	1.93	4.37	2.31	2.59
J-test	117.1	150.6	133.1	150.9	129.5	138.5
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Table 7 shows results of estimating the stochastic discount factor of the model, (40), via generalized method of moments. See notes to Table 6 for details. The top panel (A), shows estimates of the parameters using the cross-section of 25 portfolios sorted on market-equity and book-to-market from Fama and French (1993). The bottom panel (B) shows results using the cross-section of 30 industry portfolios of Fama and French (1997). Sample includes annual data from 1963:2008.

Appendix A: Solution to the model

The Hamilton-Jacobi-Bellman equation for the social planner's optimization problem is:

$$0 \quad = \quad \max_{L_I, L_C, i_C, N} \left\{ h(C,N,J) + (i_C - \delta)\, J_{K_C} K_C + \mu_A\, J_A\, A + \frac{1}{2}\, \sigma_A^2\, J_{AA}\, A^2 + \mu_Z J_Z Z + \frac{1}{2}\, \sigma_Z^2\, J_{ZZ}\, Z^2 \right\}$$

subject to $C = AK_C^{\beta_C} L_C^{1-\beta_C}$, $i_C = c^{-1}\left(\frac{Z}{K_C} K_I^{\beta_I} L_I^{1-\beta_I}\right)$ and $L_C + L_I = 1 - N$. The labor supply decision is intratemporal, since households will choose L_C to maximize CN^{ψ}. Let $\omega \equiv \ln\left(\frac{ZK_I^{\beta_I}}{K_C}\right)$ and $M_0 \equiv \left(\frac{1-\beta_C}{1+\psi-\beta_C}\right)^{1-\beta_C} \left(\frac{\psi}{1-\beta_C+\psi}\right)^{\psi}$. Guess $J = \frac{(AK_C^{\beta_C})^{1-\gamma}}{1-\gamma} f(\omega)$. When we replacing the above in the HJB equation, the first order condition with respect to L_I is

$$M_1\,(1-L_I)^{(1-\beta_C+\psi)(1-\theta^{-1})-1}\, L_I^{\beta_I} = \hat{i}'\left(e^{\omega} L_I^{1-\beta_I}\right) e^{\omega} \left[\beta_C\, + \frac{f'(\omega)}{f(\omega)} \frac{1}{\gamma-1}\right] f(\omega)^{\frac{1-\theta^{-1}}{1-\gamma}}$$

where $\hat{i}$ is the inverse function of $c(i_C)$ and $M_1 \equiv \rho((1-\beta_C+\psi))\, M_0^{1-\theta^{-1}} (1-\beta_I)^{-1} > 0$. Denoting by $L_I(\omega)$ as the solution to the equation above, we can characterize $f(\omega)$ as the solution to the following ordinary differential equation

$$\begin{aligned} 0 \quad &= \quad \rho \frac{1-\gamma}{1-\theta^{-1}} \left(\frac{(1-L_I(\omega))^{(1-\beta_C+\psi)(1-\theta^{-1})}\, M_0^{1-\theta^{-1}}}{(f(\omega))^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} \right) + f'(\omega) \left(\delta + \mu_Z - \frac{1}{2}\sigma_Z^2 - \hat{i}\left(e^{\omega} L_I(\omega)^{1-\beta_I}\right) \right) + f''(\omega) \frac{1}{2} \sigma_Z^2 \\ &- \quad f(\omega) \left(\frac{\rho(1-\gamma)}{1-\theta^{-1}} + \beta_C\,(\gamma-1)\,\hat{i}\left(e^{\omega} L_I(\omega)^{1-\beta_I}\right) - \delta\,\beta_C(\gamma-1) + \mu_A\,(\gamma-1) - \frac{1}{2}\sigma_A^2 \gamma(\gamma-1) \right) \end{aligned}$$

An application of Ito's Lemma on π_t (given by equation 18) allows us to recover the instantaneous risk-free rate r_f and the prices of risk b_Z and b_A:

$$\begin{aligned} \frac{d\pi_t}{\pi_t} \quad &= \quad -\underbrace{\left\{ -h_J(\omega_t) + \gamma\mu_A - \frac{1}{2}\gamma(\gamma+1)\sigma_A^2 + \beta_C \gamma (i_C(\omega_t) - \delta) - \frac{\phi'(\omega_t)}{\phi(\omega_t)} \left[(\mu_Z - \frac{1}{2}\sigma_Z^2) - i_C(\omega_t) + \delta \right] - \frac{1}{2} \frac{\phi''(\omega_t)}{\phi(\omega_t)} \sigma_Z^2 \right\}}_{r_{f,t}} \\ &- \underbrace{\gamma\sigma_A}_{b_A} dB_t^A - \underbrace{-\frac{\phi'(\omega_t)}{\phi(\omega_t)} \sigma_Z}_{b_Z(\omega_t)} dB_t^Z, \end{aligned}$$

where

$$\begin{aligned} h_J(\omega) \quad &= \quad -\rho \frac{\gamma-\theta^{-1}}{1-\theta^{-1}} M_0^{1-\theta^{-1}} (1-L_I(\omega))^{(1-\beta_C+\psi)(1-\theta^{-1})} (f(\omega))^{-\frac{1-\theta^{-1}}{1-\gamma}} - \rho \frac{1-\gamma}{1-\theta^{-1}} \\ \phi(\omega) \quad &\equiv \quad (1-L_I(\omega))^{-\theta^{-1}(1-\beta_C)+\psi(1-\theta^{-1})} (f(\omega))^{-\frac{\gamma-\theta^{-1}}{1-\gamma}}. \end{aligned}$$

The value of a consumption firm can be computed as follows: the firm buys new capital at price p^I and hires labor L_C to maximize its value

$$\begin{aligned} \pi_t S_t^C \quad &= \quad E_t \int_t^{\infty} \max_{L_{C,s}, i_{C,s}} \pi_s \left(A_s K_{C,s}^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} - p_s^I\, Z_{m,s}^{-1}\, c(i_C) K_C \right) \\ S_t^C \quad &= \quad E_t \int_t^{\infty} \exp\left(\int_t^s h_J(C_u, N_u, J_u)\, du \right) \frac{h_{C,s}}{h_{C,t}} \left(\beta_C A_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - p_s^I\, Z_{m,s}^{-1}\, c(i_{C,s}^*) K_{C,s} \right) ds \end{aligned}$$

The planner's Lagrangian evaluated at the optimum $(L^*_{C,s}, i^*_{C,s})$ can be written as:

$$\mathcal{L}_t = E_t \int_t^\infty h(C^*_s, N^*_s, J^*_s) - \pi_s(C_s - A_s K_{C,s}^{\beta_C} {L^*_{C,s}}^{1-\beta_C}) - p^I_s \pi_s \left(Z^{-1}_{m,s}\, c(i^*_{C,s}) K_{C,s} - Z_{I,s} K_I^{\beta_I} {L^*_I}^{1-\beta_I} \right) ds$$

Given that $\frac{\partial \mathcal{L}}{\partial K_C} = \frac{\partial J}{\partial K_C}$, an application of the envelope theorem yields

$$\frac{\partial J_t}{\partial K_{C,t}} K_{C,t} = E_t \int_t^\infty \exp\left(\int_t^s h_J(C_u, N_u, J_u)\, du \right) h_{C,s} \left(\beta_C A_s K_{C,s}^{\beta_C} (L^*_{C,s})^{1-\beta_C} - p^I_s\, Z^{-1}_{m,s}\, c(i^*_{C,s}) K_{C,s} \right) ds$$

The above, along with a similar calculation for the investment sector implies that

$$S^C_t = \frac{1}{h_{C,t}} \frac{\partial J_t}{\partial K_{C,t}} K_{C,t} \qquad S^I_t = \frac{1}{h_{C,t}} \beta_I J_Z Z_t.$$

The sensitivity of the investment- and consumption-goods sector to the investment shock equal

$$\zeta_I(\omega_t) = \frac{f''(\omega_t)}{f'(\omega_t)} - \frac{\phi'(\omega_t)}{\phi(\omega_t)} \quad \text{and} \quad \zeta_C(\omega_t) = \frac{\beta_C(1-\gamma) f'(\omega_t) - f''(\omega_t)}{\beta_C(1-\gamma) f(\omega_t) - f'(\omega_t)} - \frac{\phi'(\omega_t)}{\phi(\omega_t)}.$$

The market portfolio equals the sum of the investment- and consumption-goods sector. It's sensitivity to the investment shock equals

$$\zeta_M(\omega_t) = \frac{\beta_C(1-\gamma) f'(\omega_t) - (1-\beta_I)\, f''(\omega_t)}{\beta_C(1-\gamma) f(\omega_t) - (1-\beta_I)\, f'(\omega_t)} - \frac{\phi'(\omega_t)}{\phi(\omega_t)}.$$

To find the value of assets in place in the C-sector, consider the value of a firm that plans to not invest in the future

$$\pi_t S^{VC}_t = \max_{L_C} E_t \int_t^\infty \pi_s \left(A_s (K_{C,t} e^{-\delta\, s})^{\beta_C} L^{1-\beta_C}_{C,s} - w_s L_{C,s} \right) ds.$$

The optimal labor decision $\hat{L}_C$ of a firm with no new investment equals:

$$(1-\beta_C) A_s (K_{C,0} e^{-\delta\, s})^{\beta_C} \hat{L}^{1-\beta_C}_{C,s} = w_s \hat{L}_{C,s}$$

Let $\Lambda_{s,t} = \exp(\int_t^s i^*_{C,u}\, du)$. Now consider a typical firm who follows the optimal investment policy L^*_C. Its first order condition is

$$(1-\beta_C) A_s {K_{C,s}}^{\beta_C} {L^*_{C,s}}^{1-\beta_C} = w_s L^*_{C,s}$$

dividing through, we can express the labor choice of the 'value' firm as $\hat{L}_{C,s} = L^*_{C,s} \Lambda^{-1}_{s,t}$, which implies

$$\begin{aligned}
\pi_t S^{VC}_t &= E_t \int_t^\infty \pi_s A_s\, K^{\beta_C}_{C,t} e^{-\delta \beta_C\, s} \hat{L}^{1-\beta_C}_{C,0}\, ds \\
h_{C,t}\, S^{VC}_t &= E_t \int_t^\infty \exp\left(\int_t^s h_J(C,N,J) - i_{C,u}\, du \right) \beta_C\, h_{C,s}\, C_s\, ds \\
&= \frac{A_t^{1-\gamma} K_t^{\beta_C(1-\gamma)}}{1-\gamma} \times Et \int_t^\infty \exp\left(\int_t^s \hat{\rho}_u du \right) \rho \beta_C (1-\gamma) L_{C,s}^{(1-\beta_C)(1-\theta^{-1})} N_s^{\psi(1-\theta^{-1})} f(\omega_s)^{\frac{\gamma-\theta^{-1}}{\gamma-1}}\, ds \\
&= \frac{A_t^{1-\gamma} K_{C,t}^{\beta_C(1-\gamma)}}{1-\gamma} g(\omega_t)
\end{aligned}$$

An application of Ito's lemma on the right hand side, along with the fact that conditional expectations are a martingale imply that $g(\omega_t)$ can be computed as the solution to the ODE:

$$
\begin{aligned}
0 \quad = \quad & \rho\beta_C(1-\gamma)M_0^{1-\theta^{-1}}\left(1-L_I(\omega)\right)^{(1-\beta_C+\psi)(1-\theta^{-1})}\, f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} + g'(\omega)\left(\delta+\mu_Z-\frac{1}{2}\sigma_Z^2-\hat{i}\left(e^{\omega}L_I(\omega)^{1-\beta_I}\right)\right) \\
& +\left(h_J(\omega)+(\beta_C(1-\gamma)-1)\,\hat{i}\left(e^{\omega}L_I(\omega)^{1-\beta_I}\right)-\delta\beta_C(1-\gamma)+(1-\gamma)\left(\mu_A-\frac{1}{2}\sigma_A^2\right)+\frac{1}{2}\left(1-\gamma\right)^2\sigma_A^2\right)g(\omega) \\
& +g''(\omega)\frac{1}{2}\sigma_Z^2
\end{aligned}
$$

The sensitivities of value and growth firms on the investment shock are given by

$$
\zeta_G(\omega_t)=\frac{\beta_C(1-\gamma)f(\omega_t)-f'(\omega_t)-g(\omega_t)}{\beta_C(1-\gamma)f'(\omega_t)-f''(\omega_t)-g'(\omega_t)}-\frac{\phi'(\omega_t)}{\phi(\omega_t)} \quad \text{and} \quad \zeta_V(\omega_t)=\frac{g'(\omega_t)}{g(\omega_t)}-\frac{\phi'(\omega_t)}{\phi(\omega_t)},
$$

Appendix B: Data construction

Macroeconomic quantities

My measure of output is GDP excluding government consumption; consumption is non-durables plus services; labor supply is non-farm business hours; investment is non-residential fixed investment. I use nominal values and deflate them by the consumption deflator. I compute per-capital values by dividing by population. Data on quantities comes from the Bureau of Economic Analysis (BEA). I obtain the relative price of investment goods from Israelsen (2010), who extends the quality-adjusted investment price series constructed by Gordon (1990) and Cummins and Violante (2002) through 2008.

Investment and consumption firms

I follow Gomes et al. (2009) and classify firms as investment or consumption producers based on the U.S. Department of Commerce's National Income and Product Account (NIPA) tables. I classify industries based on the sector to which they contribute the most value. I use the 1997 Input-Output tables who map industries into investment or consumption producers based on NAICS codes.

10 IMC-beta portfolios

I estimate pre-ranking betas with the IMC portfolio using weekly data and a window of one year. I sort firms into portfolios based on their pre-ranking beta, and rebalance portfolios at the end of every December.

10 book-to-market sorted portfolios

I sort firms into 10 portfolios based on the ratio of book-equity (COMPUSTAT item ceq) to market capitalization (CRSP item prc times shrout) in December. I sort firms into 10 portfolios using NYSE breakpoints and rebalance portfolios every June. I exclude firms in the investment sector.

25 market capitalization and book-to-market sorted portfolios

Available through Kenneth French's website.

30 industry portfolios

Available through Kenneth French's website.